Non-circular gears

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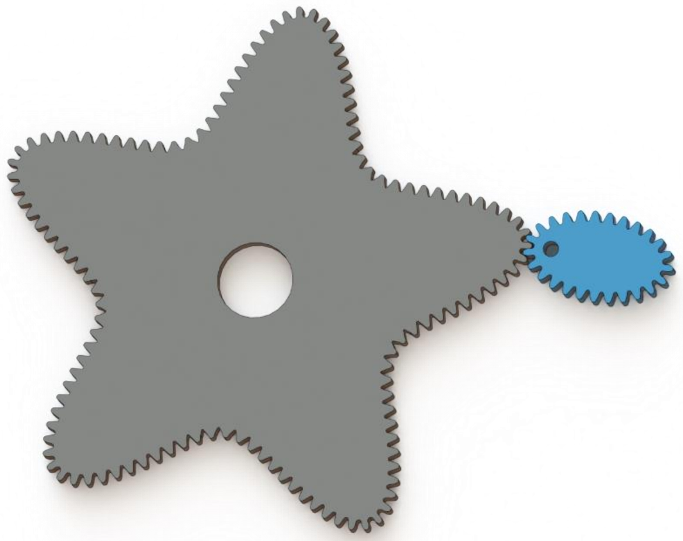


Abstract

Non-circular gears (NCGs) were first sketched by Leonardo da Vinci around 1500 and have since found their way into many applications. The circular gears are usually designed to transmit constant torque with maximum efficiency and minimum noise. On the contrary, the non-circular gears can be designed to obtain variable transmission ratios (and thus varying loads and speeds) and/or varying centre distances.

Due to the NCG's manufacturing challenges in traditional gear materials (like steel or brass), they are used in smaller applications, where gears are manufactured with processes like injection moulding or sintering. This way, the tooth form is not limited by the hobbing cutter geometry.
The non-circular gears are used in textile industry to improve kinematics, in potentiometers, in mechanical presses, in flow meters and also in small actuators.

The paper will present the basic kinematics behind the non-circular gears, the calculation of the operating pitch lines as well as the tooth generation. As the strength calculation of non-circular gears is not standardized, a special calculation method is needed that enables an approximate strength calculation of the non-circular gears.

Keywords:

Non-circular gears, design principles, kinematics, strength calculation

1. Introduction

Non-circular gears (NCGs) were first sketched by Leonardo da Vinci around 1500 and have since found their way into several applications. First known publications came in the 19th century by Holditch, Brown and Reuleaux [1-3]. Later, several relevant publications followed [4-8]. In the last decade, there has been an increased interest in the field of non-circular gears [9-11] due to their advantages over circular gears. A comprehensive overview of the non-circular gears was done by Litvin [12]. Even though more and more publications are available on non-circular gears, the knowledge is, especially compared to cylindrical gears, still very limited.

The non-circular gears are designed to have variable output loads and speeds. It is also possible to design them with variable center distances. Usually, there are two possibilities for designing the non-circular gears:
1) The operating pitch line (OPL) of the driving gear is known and the transmission ratio function and the OPL of the driven gear are calculated,
2) The transmission ratio function is given and the OPLs of the driving and driven gears are calculated.

The operating pitch lines of non-circular gears are commonly ellipses because of their simplicity [12]. However, to achieve more complex transmission ratio functions, modified elliptical shapes, n-lobbed ellipses or other arbitrary functions can be used for the OPLs of the gears.

Due to the manufacturing difficulties of the non-circular gears in traditional gear materials (like steel or brass), they are finding their way into smaller applications, where gears are manufactured with injection moulding or sintering. The price of manufacturing a non-circular gear in plastic is similar as manufacturing a similar plastic cylindrical gear.

The variable transmission function is the main advantage of the non-circular gears (compared to the cylindrical gears). However, additional vibrations and bearing forces can occur because of variable loads and speeds, especially if gears are not rotating in the mass center.

They are often used in the textile industry to improve kinematics, in potentiometers, in CVTs, in mechanical presses, and in steering applications for cars and ships [7, 13, 14].

This paper will present the basic kinematics behind the non-circular gears, the calculation of the operating pitch lines as well as the tooth generation. As the strength calculation of non-circular gears is not standardized, a special calculation method will be shown that enables an approximate strength calculation of the non-circular gears.

2. Operating pitch lines

Non-circular gears can be simply represented by 2 operating pitch lines. According to the law of gear meshing, the operating pitch lines must roll together without slipping. Figure 1 shows the operating pitch lines, representing the corresponding non-circular gears.

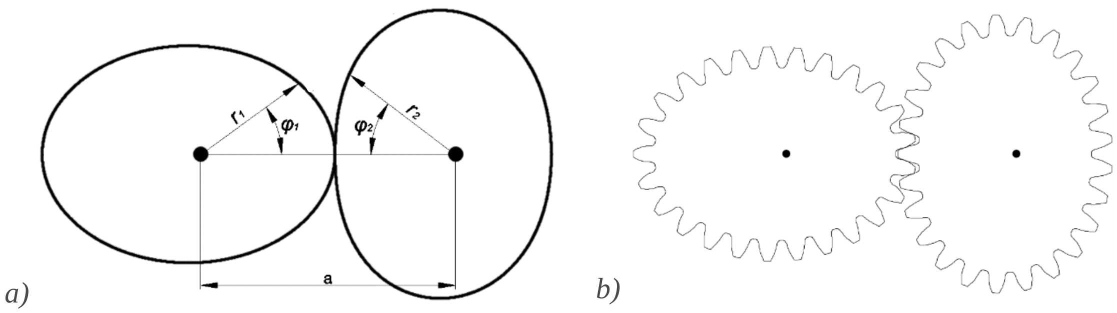


Figure 1. a) the operating pitch lines and b) the corresponding gears.

The operating pitch lines from Figure 1 can mathematically be represented in the polar coordinate systems by $r_1(\varphi_1)$ and $r_2(\varphi_2)$. The corresponding transmission ratio function i of the gears can be written as

$$i(\varphi_1) = \frac{\omega_1(\varphi_1)}{\omega_2(\varphi_2)} = \frac{r_2(\varphi_2)}{r_1(\varphi_1)}, \qquad \text{Eq. (1)}$$

where ω_i is the gear angular velocity of a gear.

From Figure 1, the center distance a can be written as

$$r_1(\varphi_1) + r_2(\varphi_2) = a. \qquad \text{Eq. (2)}$$

For this derivation, the center distance a is assumed to be constant, however it can also be defined as a function of the rotation angle – $a(\varphi_1)$.

Using eq. 1 and eq. 2, the transmission ratio function i can be rewritten to

$$i(\varphi_1) = \frac{a - r_1(\varphi_1)}{r_1(\varphi_1)}. \qquad \text{Eq. (3)}$$

To fulfill the no-slip rolling conditions, the tangential velocities at the point of contact must be equal and perpendicular to the line connecting the centers of rotation. Therefore

$$r_1(\varphi_1)\, d\varphi_1 = r_2(\varphi_2)\, d\varphi_2. \qquad \text{Eq. (4)}$$

For calculation of the rotation angle φ_2, two different cases can be considered:

CASE 1: If $r_1(\varphi_1)$ is given, eq. 4 for can be solved for φ_2 (using eq. 1 and 3):

$$d\varphi_2 = \frac{r_1(\varphi_1)}{r_2(\varphi_2)} d\varphi_1$$
$$d\varphi_2 = \frac{r_1(\varphi_1)}{a - r_1(\varphi_1)}\, d\varphi_1 \qquad \text{Eq. (5)}$$
$$\varphi_2 = \int_0^{\varphi_1} \frac{r_1(\varphi_1)}{a - r_1(\varphi_1)}\, d\varphi_1.$$

CASE 2: If $i(\varphi_1)$ is given ($r_1(\varphi_1)$ and $r_2(\varphi_2)$ are not known), eq. 4 (using eq. 1) can be rewritten to

$$d\varphi_2 = \frac{r_1(\varphi_1)}{r_2(\varphi_2)} d\varphi_1$$
$$d\varphi_2 = \frac{1}{i(\varphi_1)}\, d\varphi_1 \qquad \text{Eq. (6)}$$
$$\varphi_2 = \int_0^{\varphi_1} \frac{1}{i(\varphi_1)}\, d\varphi_1.$$

Using either eq. 5 or eq. 6, a relation between φ_1 and $\varphi_2(\varphi_1)$ can be calculated.

To have continuous rotation of the gears, the operating pitch lines must be closed centrodes. The transmission ratio function $i(\varphi)$ must be a periodic function and its period T must be related with the periods of revolution of each individual gear T_1 and T_2:

$$T = \frac{T_1}{n_2} = \frac{T_2}{n_1}, \qquad \text{Eq. (7)}$$

where n_1 and n_2 are Z^+.

Several examples of the closed centrodes are shown in Figure 2. It is possible to have two identical non-circular gears in contact (2a), two different non-circular gears (2b, 2c, 2d), a non-circular n-gear chain drive (2e) or an internal non-circular gear (2f).

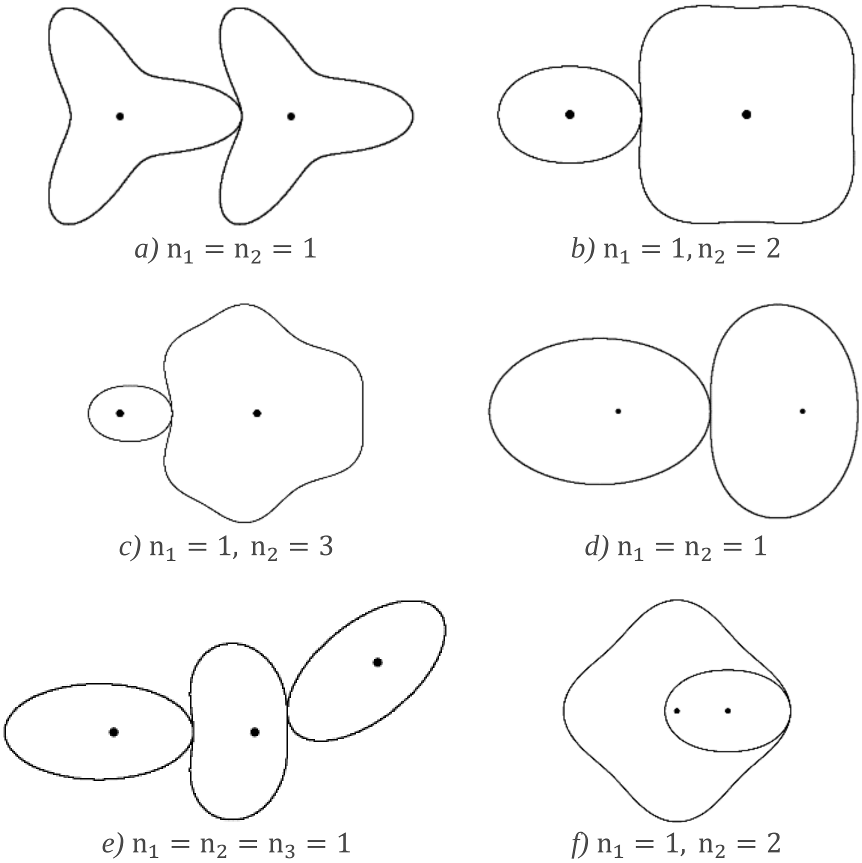


Figure 2. Examples of closed operating pitch lines.

If the gears are used for non-continuous rotation, then additional conditions from eq. 7 are not required. However, in case of unclosed centrodes, the angular rotation of the engaging gears is limited. Examples of unclosed centrodes of 2 non-circular gears are shown in Figure 3a, while Figure 3b shows a rack and pinion configuration.

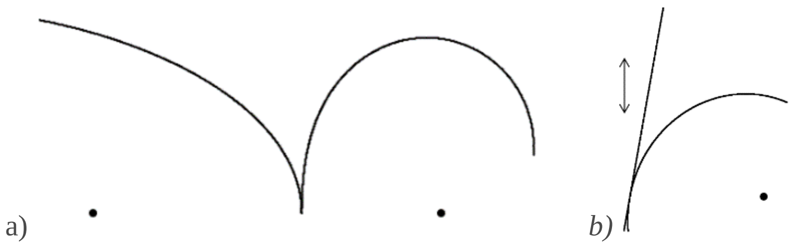


Figure 3. Examples of unclosed operating pitch lines.

In most cases, ellipses are used as operating pitch lines for the closed centrodes. However, also other functions (circles, modified ellipses [12], n-lobbed generalized ellipses [10] or own shapes) can be used.

2.1. Adding eccentricity to the rotation point

An offset (in both *x* and *y* direction) can be applied to the center of rotation of the driving gear. Figure 4 shows the operating pitch lines for different *x* and *y* offsets of the rotation point of the driving ellipse. The semi-minor axis b_e of the driving ellipse had to be adjusted for selected offset to fulfill conditions from eq. 7. However, the differences between operating pitch lines are still minor.

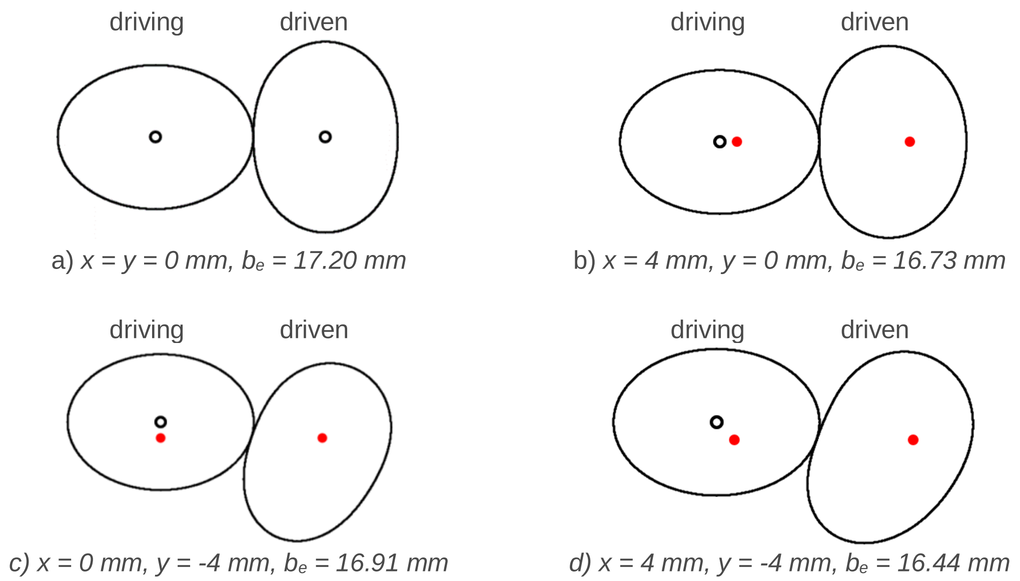


Figure 4. Operating pitch lines for different offset of the rotation point (rotation points are marked with red, original rotations point with black dot).

Figure 5 shows transmission ratio functions calculated from the operating pitch lines from Figure 4. A uniform transmission function is obtained when no rotation point offset is applied to the driving gear. Significant changes in the transmission ratio functions can be generated using offsets of the rotation point in different directions.

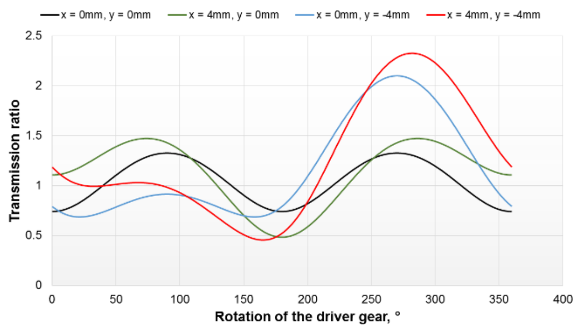
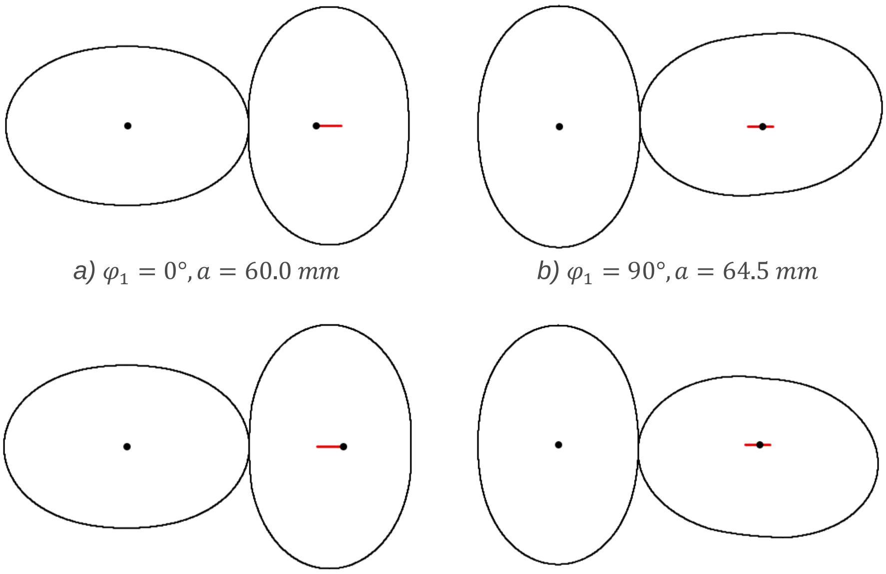


Figure 5. Variations in transmission ratio due to the eccentricity of the rotation point.

Given an infinite number of possibilities of different operating pitch line curves, center distances and rotation offsets, it is possible to fit the desired transmission ratio function precisely.

2.2 Variation of transmission ratio and center distance

Non-circular gears can have an additional feature that can simplify the product's design. Besides variable transmission ratio, non-circular gears also have the possibility to adjust the center distance during rotation simultaneously (assuming meshing on both flanks). An example of rotation is shown in Figure 6. The center distance function is shown in Figure 7.



a) $\varphi_1 = 0°, a = 60.0\ mm$ b) $\varphi_1 = 90°, a = 64.5\ mm$

c) $\varphi_1 = 180°, a = 68.0\ mm$ d) $\varphi_1 = 270°, \mathrm{a} = 64.5\ \mathrm{mm}$

Figure 6. Rotation of NCG's with varying center distance.

Variation of the transmission ratio and the center distance with the rotation angle of the driving gear is shown in Figure 7. The center distance function can be an arbitrary function; however in continuous rotation, the start and end value must be the same.

Figure 7. Transmission ratio and center distance variation

2.3 Non-circular pinion and rack configuration

It is also possible to design a non-circular pinion and rack configuration. Examples with closed and unclosed centrodes are shown in Figure 8. The design can incorporate circular pinion (offset of the rotation point), which is meshing with a non-straight rack (Figure 8a), a non-circular pinion meshing with a non-straight rack (8b) or a non-circular pinion meshing with a straight (inclined) rack (8c).

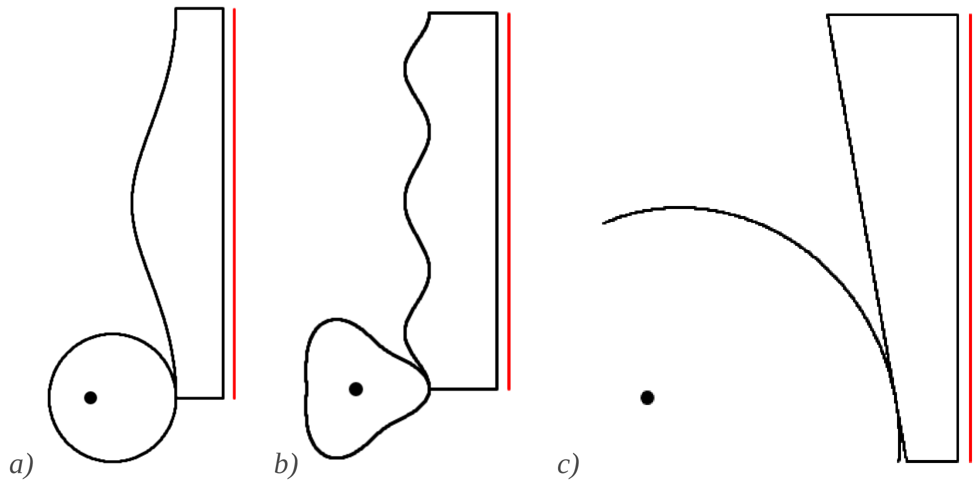


Figure 8. a) circular pinion and non-straight rack, b) non-circular pinion and non-straight rack and c) non-circular pinion and straight (inclined) rack.

Variation of the tangential force and lateral movement of the rack for example a and c from Figure 8 are shown in Figure 9.

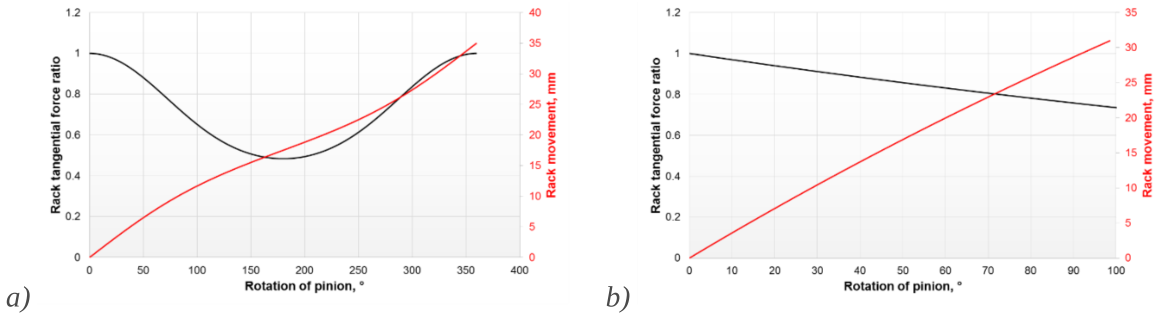


Figure 9. Tangential force and movement of the rack for a) circular pinion/non straight rack and b) non-circular pinion/straight (inclined) rack.

3. Generation of the tooth form

The tooth form of the non-circular gear can be generated by a reference rack, as shown in Figure 10. To generate the tooth form, the reference line of the rack must roll without slipping on the operating pitch line of the non-circular gear.

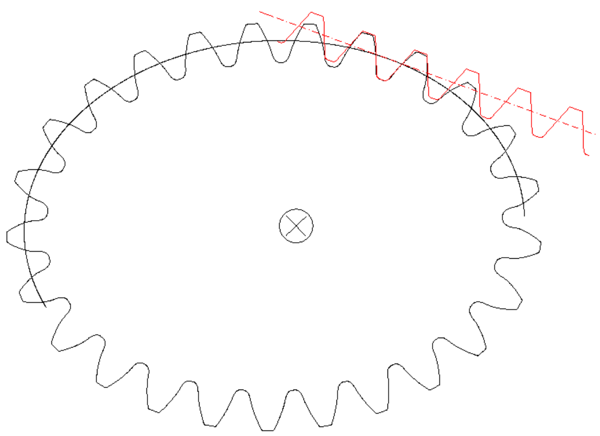


Figure 10. Generating tooth form in the non-circular gear using the reference rack.

However, only outer non-circular gears can be produced using a reference rack. To produce the inner non-circular gears, a pinion type cutter must be used, which is shown in Figure 11. A pinion type cutter can also be used to produce outer gears.

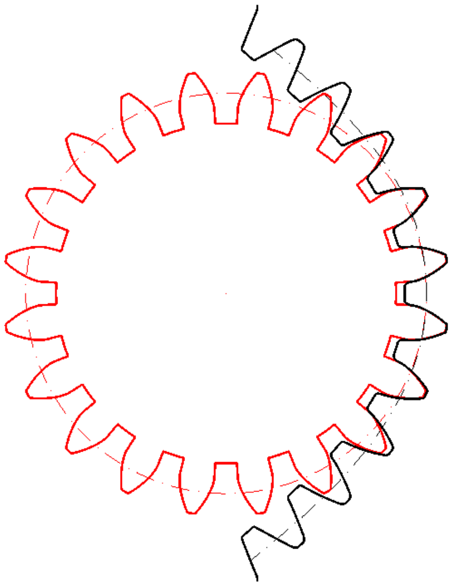


Figure 11. Pinion type cutter (red) and generated internal non-circular gear (black).

More about the generation of the non-circular gears can be found in the literature [12, 16].

4. Strength calculation of non-circular gears

Currently, there is no available standard or guideline for the strength calculation (root and flank) of the non-circular gears. However, it is still possible to estimate the strength of the non-circular gears. Two different approaches can be used: finite element analysis (FEM) or calculation with the replacement cylindrical gear [15]. FEM can be very time-consuming, as multiple meshing positions must be checked. In this paper, calculation with the replacement gear will be discussed.

A non-circular gear in a defined, stationary state, can be represented by a temporary replacement cylindrical gear. To completely analyze the non-circular gear, several individual teeth must be analyzed. Several criteria can be used to select the teeth for further analysis: the tooth with the max. tangential force, the tooth with the smallest thickness at the root (most undercut) or the tooth that is nominally the most loaded (load to root thickness/rounding ratio). Also, other criteria can be used.

Figure 12 shows the non-circular gears for strength evaluation together with the tooth on the first gear, which is selected for further analysis.

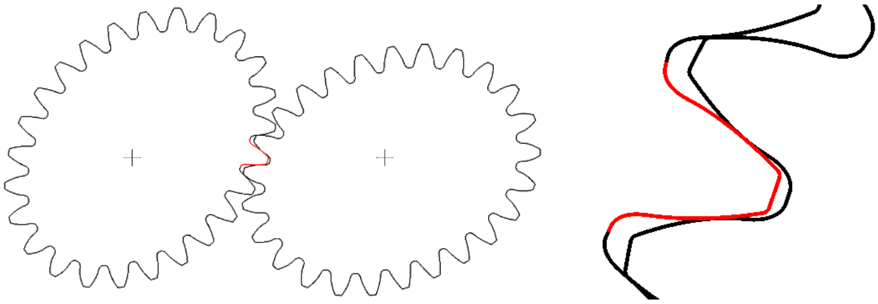


Figure 12. Non-circular gear with the selected tooth for the strength evaluation.

Figure 13 shows the non-circular gears and their replacement diameters. The replacement diameters d_1 and d_2 are calculated from the operating pitch line curvatures at the contact point. If the operating pitch lines are functions (i.e. ellipses), then the curvature radii can be calculated with an exact formula. However, if the operating pitch lines are approximated with points, a 3 (or more) point approximation around the contact point can be used to calculate replacement diameters and center points. Once the replacement diameters are calculated, a replacement center distance can be calculated as $a_r = d_1 + d_2$. To construct the replacement gears, the tip diameter d_a and the root diameter d_f must also be determined (Figure 13).

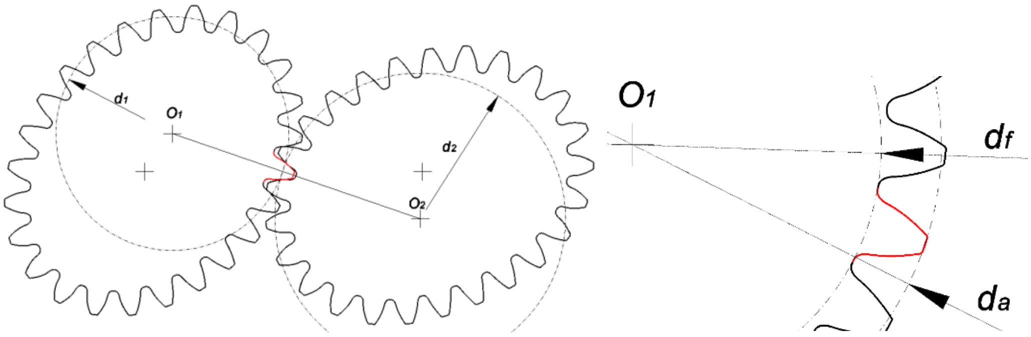


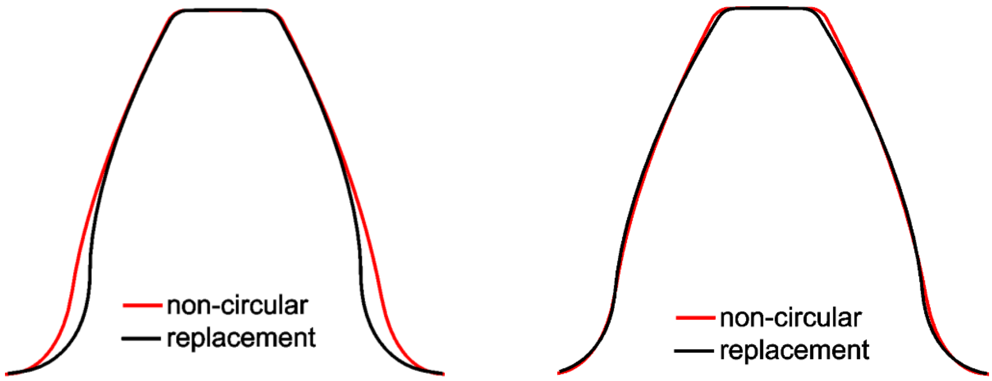
Figure 13. Reference, tip and root diameters of the replacement gear of the driving gear.

The number of teeth of the replacement gears (z_R) is calculated using eq. 7. The normal module should be the same as used for the non-circular gears. For the first approximation, also the same pressure angles α_n can be used.

$$z_{R,i} = \frac{d_i}{m_n}. \qquad \text{Eq. (7)}$$

With all the necessary parameters of the replacement gear known, KISSsoft can be used to construct and calculate the replacement gear. To have a relevant strength analysis, the tooth shape of the non-circular and the replacement gears must match as best as possible, especially in the root area.

Figure 14 shows comparison of the tooth form (calculated in KISSsoft) before and after the parameter optimization. A close match between the non-circular tooth and the replacement gear tooth can be obtained by adjusting pressure angle, reference profile parameters and tooth thickness tolerances.



a) m_n = 1.6123 mm, α = 20°
ρ_{fp} = 0.61 mm, A_{sn} = -0.05 mm

b) m_n = 1.6123 mm, α = 23°
ρ_{fp} = 0.90 mm, A_{sn} = 0.00 mm

Figure 14. Tooth form comparison a) before and b) after optimization.

In special cases, when this method does not give good results, it is possible in KISSsoft to export the tooth form from a non-circular gear module as a .dxf and import it into the replacement gear calculation.

With the replacement gear geometry determined (Figure 14), a normal gear calculation can be carried out to estimate the lifetime and safety factors of the non-circular gears. However, the tangential force must be recalculated (eq. 8) based on the replacement gears' new center points O_1 and O_2 (Figure 15).

$$F_{T,R} = F_T / cos\theta. \qquad \text{Eq. (8)}$$

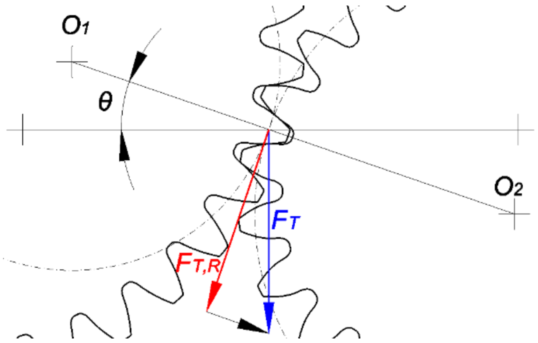


Figure 15. Tangential force for the replacement gear calculation.

In addition to the correction of the force, also the rotational speed of the replacement gear needs to be adjusted. The rotational speed of the replacement gear can be calculated using eq. 9 and is based on equal tangential velocities, as shown in Figure 16.

$$n_R = \frac{r \cdot n \cdot cos\theta}{r_R} \qquad \text{Eq. (9)}$$

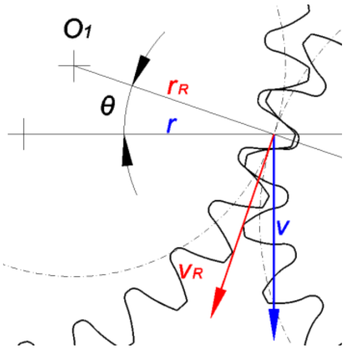


Figure 16. Rotational speed for the replacement gear calculation.

5. Conclusions

Non-circular gears are used in applications, where variable transmission ratios are desired. With the use of injection moulding or sintering, the non-circular gears can be produced without a significant price increase and are thus very suitable for serial production.

To design non-circular gears, first the operating pitch lines must be calculated. According to the gearing law, the gear's operating pitch lines must roll without slip.

Non-circular gears can be used to continuously transmit torque (operating pitch lines must be closed centrodes) or can transmit torque only within a certain rotation angle (operating pitch lines can be unclosed centrodes).

With an endless number of operating pitch lines available, combined with the possible eccentricity of the rotation points and a possible variation of center distance, it is almost always possible to match the desired transmission ratio function.

The non-circular shape can be applied to several different gear types: external and internal cylindrical gear, rack and pinion, face gear and even bevel gear.

As there is currently no valid standard available for strength evaluation of non-circular gears, a replacement gear calculation is explained. With the replacement gear method, a non-circular gear (in a defined, stationary state) can be momentarily represented by a cylindrical gear, for which a standardized strength calculation is available.

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