

# Reliability, Lifetime and Safety Factors

Dr. Stefan Beermann

*(The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.)*

## Introduction

The most important criteria for the design of a gearbox is a sufficient strength of all components. There are, however, different ways to define this demand. The two most common ones are either defining minimum required safety factors for a given lifetime or prescribing a minimum likelihood to achieve a certain lifetime, often expressed in the reliability of a component within a given lifetime. This paper discusses the different approaches and the relationship between the safety factors and the calculation of the reliabilities. It will concentrate on ISO and AGMA standards for gears, shafts and bearings and will only discuss endurance calculations, no static calculations.

After an introduction to the concept of reliability calculation based on the book of Bernd Bertsche (Ref.2), an example to show the difference between safety factor and reliability is given. After that, the built-in reliability coefficients of ISO 281 and AGMA 2001-D04 are compared to the general approach in Bertsche.

## Symbols

Symbol	Notation	Unit
$R$	Reliability (of a single component)	-
$t$	Lifetime/number of load cycles (depends on context)	h/-
$t_0$	Number of load cycles without failure	-
$T$	Characteristic service life (in cycles) with 63.2% probability of failure (or 36.8% reliability)	-
$b$	Weibull form parameter	
$f_{tB}$	Factor according to Table 2	
$L_p$	Achievable service life of the component with a failure probability $p$	h
$L_{10}$	Achievable service life of the component with 0.1 (10%) probability of failure	h
$p$	Specific probability of failure	-
$A_1$	Reliability factor from ISO 281	-
$K_R$	Reliability factor from AGMA 2001	-

## The Art of Designing a Gearbox

When challenged with the job of designing a new gearbox, the engineer has several suitable calculation methods available for the sizing of the components. Typically, these methods determine the maximum effective stress in the component and the permissible stress for the current case. The detail level of modeling can be very different, ranging from simple assumptions to sophisticated models. In most cases, the methods deliver a safety

factor in the end, which is the quotient of permissible stress over effective stress.

Due to commercial demands (cost reduction and sales increase), the sizing process has a design lifetime as a required parameter in its center. Ideally, all components should fail at the same time. Since the failure of the first critical component normally determines the end of life of the complete gear box, each component that is designed for a longer lifetime is, in this sense, overdesigned and generates unnecessary costs. For consumer products, there might be additional requirements to reduce the lifetime of the product to sell replacements.

Inside the methods, the parameter lifetime influences the permissible stress by making it dependent on the number of load cycles. This follows the idea that the damage to a part is caused by the change in stress and leads to S-N curves for the selected materials. With this, the safety factor depends on the number of load cycles.

With this procedure at hand, everything seems well-defined, and indeed in practice, this approach has worked very well. However, expecting the components to fail at the exact number of load cycles defined for the lifetime means asking too much. The S-N curves for a specific material are based on tests conducted. In these tests, samples are exposed to alternating load and the number of load cycles until failure is recorded. Of course, the results show a certain variation. The final S-N curve is therefore a statistically extracted curve for a given failure probability. Several standards define the procedure on how to perform this extraction, for instance (Ref. 11).

For the sake of clarity, we will first define the central variables.

**Design life, achievable life.** In this paper, we are only looking at fatigue strength due to changing stresses. If there are changes in stress, there are also load cycles. Typically, there are a number of hours given, which is the planned lifetime for the component or the machine. Since questions might arise on how to interpret this number (percentage of up-time considered, changing speeds), it is a good idea to transfer the hours into load cycles. In this way we end up with a number of load cycles the machine is designed for — the design life. And we might determine the maximum number of load cycles until the machine fails with a certain likelihood — this is the achievable life.

**Effective stress; permissible stress; safety factor.** Due to the loads applied, there is a stress distribution inside the components. This stress is time-dependent, changing with the load cycles. Typically, the maximum stress is calculated with a constant part (mean value) and a transient part (amplitude). For endurance, only the amplitude of the stress is relevant. The stress used for the strength assessment is called “effective stress.”

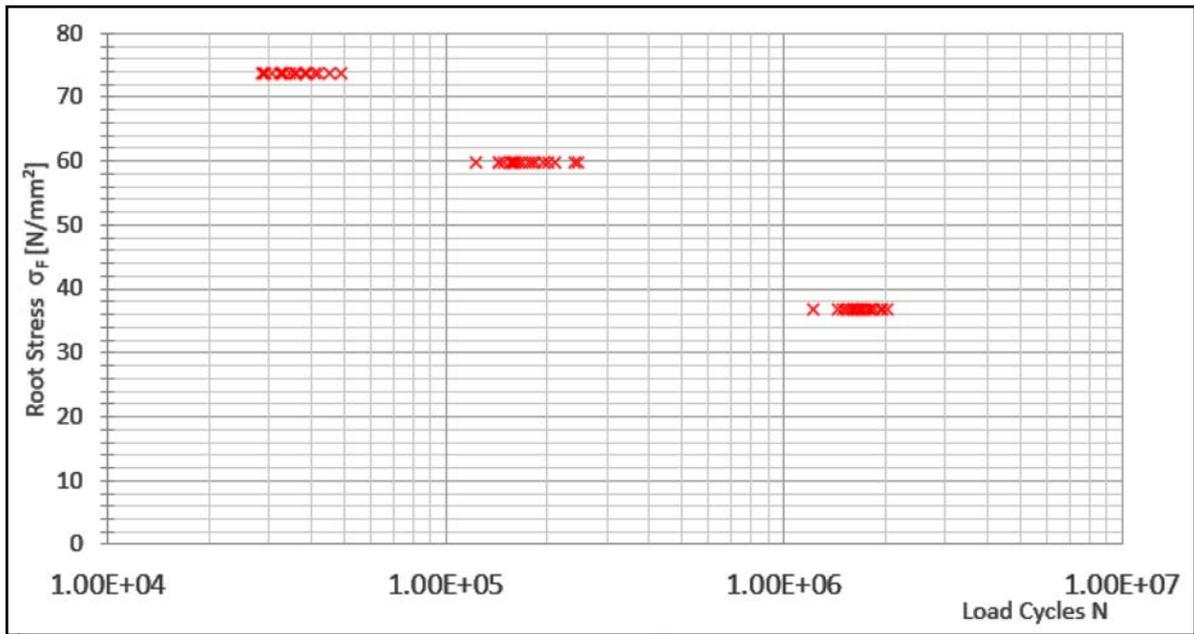


Figure 1 Series of gear tests for three different torque levels.

On the other hand, the material of the component can endure a maximum stress level for a given number of load cycles, i.e. — the “permissible stress.” The quotient of permissible stress over effective stress gives the safety factor. This safety factor must be larger than a threshold value to fulfill the requirements. This threshold value is called the “required safety factor.”

**S-N curve.** The basis of most methods is an S-N curve that defines the permissible stress limit over the number of load cycles. The name “S-N curve” simply comes from the fact that it shows a stress (S) over the number of load cycles (N).

The basis for this curve is a series of tests in which test specimens were subjected to load under standardized conditions until they failed. The number of load cycles until failure at a given load level is recorded and represents the result of one of these tests. However, if a test is repeated several times with the same load conditions, and all other environmental parameters fixed, still no one would expect all specimens to fail at the exact same number of load cycles. Rather, there will be some scattering of the results. So, some statistical evaluation needs to be done to produce an S-N curve. Figure 1 shows a typical result of a gear test with constant torque levels and the resulting scattering of load cycles until failure.

For bearings, the scattering can get quite extreme. Figure 2 shows a graph from Harris (Ref. 1). On the y-axis, the number of load cycles is found; on the x-axis is the percentage of failed bearings. The first bearing fails after about  $30 \cdot 10^6$  revolutions, and the last one after  $1,800 \cdot 10^6$  revolutions. The result is a factor of 60 from the first to the last! The  $L_{10}$  life-time in this case — where 10% of the bearings failed and 90% are still working — would be about  $120 \cdot 10^6$  revolutions. This is about 4 times more than the first failure and 15 times less than the last one.

Assuming an arbitrary number of tests were conducted to allow statistical evaluation, the combination of all tests at a specific load level requires the definition of a

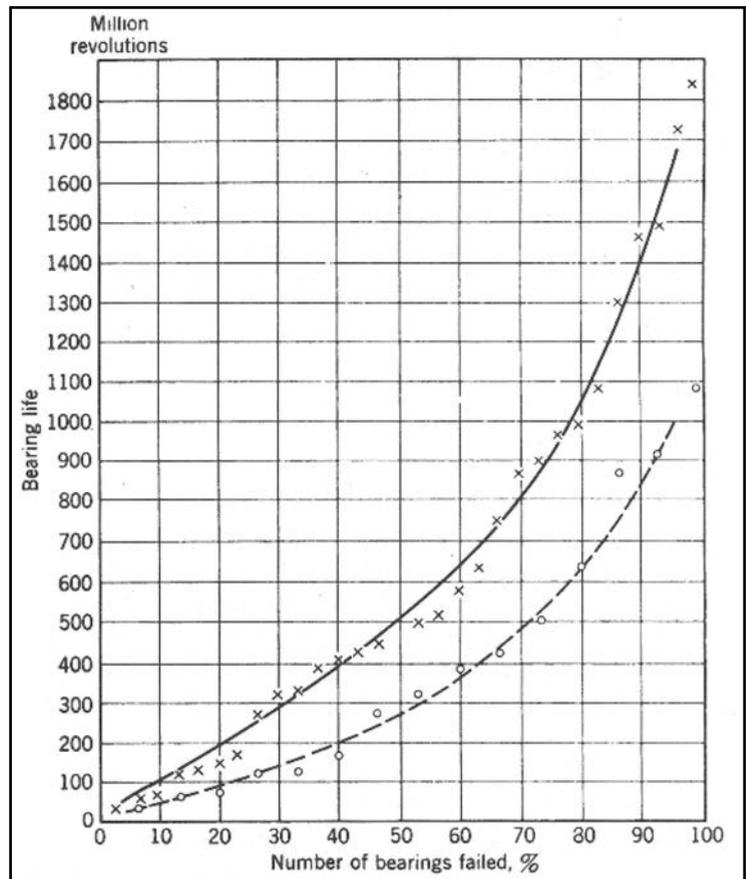


Figure 2 Results of bearing tests (Ref. 1).

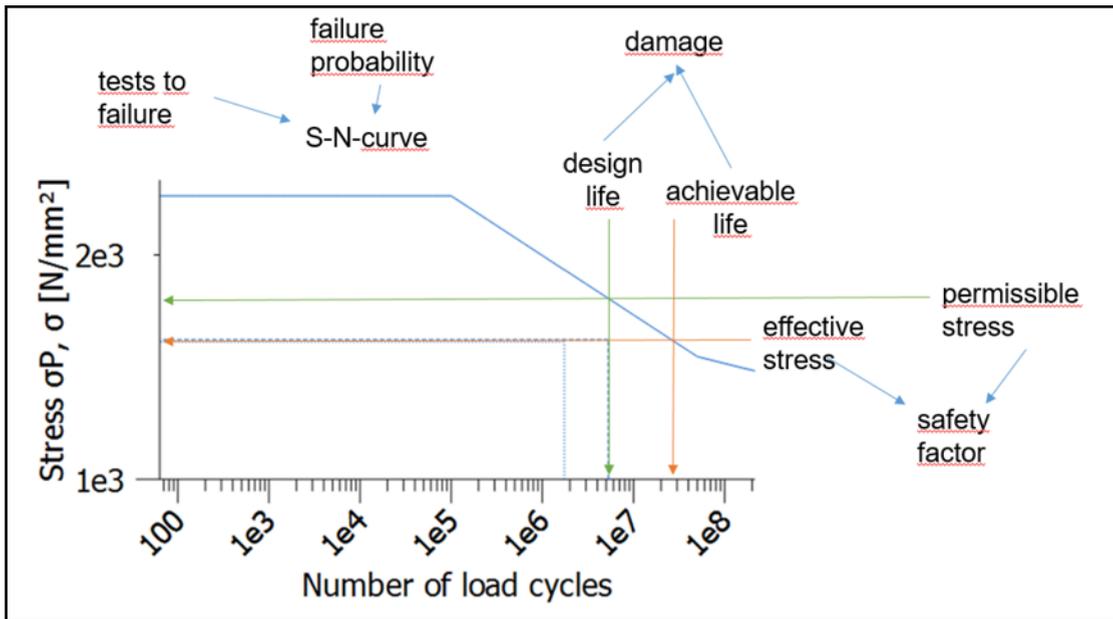


Figure 3 Relationships between strength parameters.

probability value. Changing the probability shifts the S-N curve horizontally.

**Damage.** The calculation methods discussed here all follow the concept of damage accumulation. This assumes that small cracks or failures in the material structure are enlarged due to the changing stress levels. The theory predicts the growth of the crack following a logarithmic law. The ratio of design life over achievable life is called “damage.” The idea is that the same length on the load cycle’s axis (which is scaled logarithmically) corresponds to the same amount of damage caused. The damage is usually expressed as a percentage, with the idea that reaching 100% damage means failure. Mathematically, damage larger than 100% is possible.

**Relationships.** Figure 3 shows the relationships between the terms described above. It starts with an appropriate number of tests, which in conjunction with a given failure probability, lead to an S-N-curve. The curve defines the permissible stress over the number of load cycles, so if we use the graph with a given number of load cycles (design life), we find the corresponding permissible stress (green arrows). On the other hand, if we have a given effective stress, then we can read the achievable life from the diagram (orange arrows). The quotient of permissible stress over effective stress gives the safety factor. And finally, the ratio of design life to achievable life defines the damage.

### Methods to Dimension a Gearbox

There are different approaches used for designing a gearbox. The simplest one is to determine the stresses in the components and make sure to stay below a given threshold that comes from experience. This approach is not very sophisticated, since it blanks out many influences that affect the permissible stress. Still, it is commonly used, especially when FEMs (finite element methods) are applied, simply because an FEM can only

calculate the effective stress — not the permissible.

An alternative is the application of a standardized method (or textbook method). The most common concept here is the determination of the effective stress by applying simplified models leading to analytical formulations. In a second step, the permissible stress is calculated (or read from a table) and compared to the effective stress by calculating the safety factor. Instead of a safety factor, some methods provide the exposure of the material. This is basically the same, only expressed in a different way. As a design requirement, a minimum safety factor (or maximum exposure) is given.

Some of the standardized methods for bearings calculation, like ISO 281 (Ref. 3), directly deliver an achievable life out of the loads. In the case of the bearings, the loads are the forces in radial and axial direction.

If load spectra are applied, a different approach is the calculation of damage. The advantage is that damage is more easily compared across different components than safety factors.

### Failure Probability of Machine Elements

All the above methods have one major weakness in common: they are based on an intrinsic failure probability which differs from method to method (Table 1). Therefore if a shaft has a calculated safety factor of 1.2 and a gear root has a safety factor of 1.3, it is not clear which is the more critical component. As well, the calculated life of a bearing and of a gear are not directly

Table 1 Probability of failure used by various calculation methods when determining material properties (Ref. 8)		
Calculation procedure	Probability of damage $p$	Comment
Shaft, DIN 743	2.5%	Assumed, not documented
Shaft, FKM guideline	2.5%	
Shaft, AGMA 6001	1%	If $k_c = 0.817$
Bearing, ISO 281	10%	If factor $a_1 = 1.0$
Tooth flank, ISO 6336; DIN 3990	1%	
Tooth bending, ISO 6336; DIN 3990	1%	
Tooth flank, AGMA 2001	1%	If reliability factor $K_R = 1$
Tooth bending, AGMA 2001	1%	If reliability factor $K_R = 1$

comparable.

A material strength value with a failure probability of 90% is higher than a material strength value with a failure probability of 99%. So if the 90% failure probability is applied, the safety factor is greater and the element has both a greater service life and a lower damage rate for its design life. Damage that is calculated using the methods prescribing different failure probabilities cannot be compared directly. A gear unit may fail because a part that is not considered to be critical breaks prematurely. This happens quite frequently in real life.

To overcome this problem, the reliability concept can be used. Here the result is a curve that shows the probability of failure of a component or a system over the lifetime. When statistical parameters such as the scatter of results in a standard distribution are determined on the basis of measurements on probes, a probability of failure as a function of time (or cycles) can be determined using a statistical approach. The opposite of the probability of failure is called “reliability.” Therefore, since the reliability calculation takes into consideration the inherent failure probability (Table 1), the calculated reliability at design life of different parts can be compared effectively with each other. Also, at a given probability level the component with the smallest achievable life is the critical component of the system.

**Probability distributions.** In statistics, probability distributions are used to describe stochastic processes (see numerous textbooks, e.g. — Ref. 9). A probability distribution is a function that gives the likelihood of an event for a specific value of a probability variable. In our case the event is failure (or survival) and the probability variable is the number of load cycles.

The reliability function  $R(t)$  gives the probability of survival until  $t$  load cycles. The values of  $R(t)$  range between 0.0 and 1.0, often expressed in percent:  $R(t) \cdot 100\%$ . In principal,  $t$  is an integer value; however, due to the large number of load cycles (from several thousand to billions), we can treat it like a real value and use the existing theories.

For the definition of a probability distribution, the first derivative  $R'(t)$  is defined, i.e. — the so-called density. The density is a function that defines the probability of the event happening at a given number of load cycles.

The most common distribution for general purposes is the normal distribution. This distribution is defined by the mean value  $\mu$  and the standard deviation  $\sigma$ . The density of the normal distribution is symmetric to  $\mu$ . The standard deviation  $\sigma$  controls how wide the distribution is; although for small  $\sigma$  the density looks like it becomes zero with enough distance from the mean, it never actually does. So also for negative values of  $t$  there is a positive likelihood that failure occurs. Furthermore, the failure rate  $R'(t)/R(t)$  of the normal distribution is increasing over  $t$ . Due to these limitations, the normal distribution is not very often used in reliability engineering.

A more general approach is the Weibull distribution. Two variants are possible — the two-parametric and three-parametric Weibull distribution, where the two parametric is a special case of the three parametric.

The two-parametric Weibull distribution leads to the reliability function:

$$R(t) = e^{-\left(\frac{t}{T}\right)^b} \quad (1)$$

where  $T$  is the characteristic lifetime (defined by the condition  $R(T) = 0.632$ ) and  $b$  is the shape parameter.

The three-parametric Weibull distribution has  $t_0$  as a third parameter, which shifts the first occurrence of failure to the point  $t_0$  by the substitution:

$$t \rightarrow \tilde{t} - t_0 \quad (2)$$

This substitution gives the reliability function:

$$R(t) = e^{-\left(\frac{t-t_0}{T-t_0}\right)^b} \quad (3)$$

In practice, the Weibull distribution can be used to model a wide variety of real-world scenarios, with the most famous one being the “bathtub curve.” For this, three sections with individual parameters —  $t_0$ ,  $T$ , and  $b$  — are defined, the first with a monotonous decreasing failure rate, the second with a constant failure rate, and finally a third section with increasing failure rate.

**Determining the reliability of machine elements.** There are currently no mechanical engineering standards that include the calculation of probability. A classic source for this calculation is Bertsche’s book (Ref. 2), in which the possible processes have been described in great detail. Bertsche recommends the use of the 3-parameter Weibull distribution.

To simplify the writing of the equations, we define the lifetime  $L_p$  as the lifetime with reliability  $R(L_p) = (100-p)$  (%). Both  $p$  and  $R(t)$  are usually expressed in percent (%). To apply the Weibull distribution, several parameters are needed that depend on the type of component (gear, bearing, shaft). Bertsche tabulates the shape parameter  $b$  and a factor  $f_{ib}$ , that relates  $t_0$  to the lifetime with 10% failure probability,  $L_{10}$ ; see Equation 5.

With these parameters,  $T$  and  $t_0$  can be derived from the achievable life of the component,  $L_p$ , as follows (with failure probability  $p$  according to the calculation method from Table 1,  $b$  and  $f_{ib}$  from Table 2, according to Bertsche):

$$T = \left( \frac{L_p \cdot f_{ib} \cdot L_{10}}{\sqrt[b]{-\ln(1-p)}} + f_{ib} \cdot L_{10} \right) \quad (4)$$

$$t_0 = f_{ib} \cdot L_{10} \quad (5)$$

$$L_{10} = \frac{L_p}{(1-f_{ib}) \cdot \sqrt[b]{\frac{\ln(1-p)}{\ln(0.9)} + f_{ib}}} \quad (6)$$

In Table 2 the parameter  $b$  has a wide range for breakage of shafts and tooth root. Bertsche comments in his book that for  $b$ , this is due to the confidence intervals of the statistical analysis (some of the test batches were relatively small), but also because the shape parameter  $b$  depends on the stress level: the higher the stress level, the larger the shape parameter.

For  $f_{ib}$ , however, it is mainly for gears with pitting as failure mode where the interval gets large. Here Bertsche states that this is due to a small number of tests available. He also voices the

**Table 2 Factors for a Weibull distribution according to Bertsche (Ref. 2)**

Shafts	0.7 to 0.9	1.1 to 1.9
Ball bearing	0.1 to 0.3	1.1
Roller bearing	0.1 to 0.3	1.35
Tooth flank	0.4 to 0.8	1.1 to 1.5
Tooth root	0.8 to 0.95	1.2 to 2.2

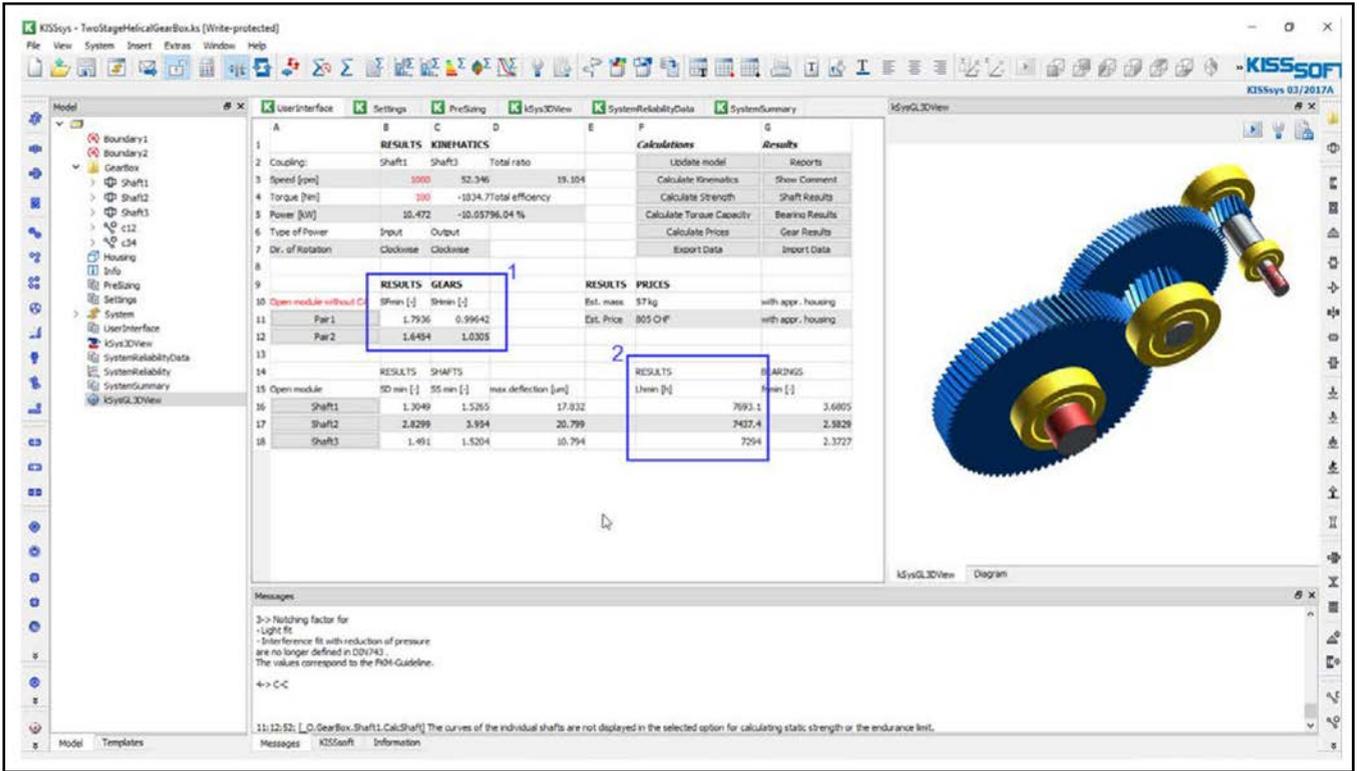


Figure 4 Example gearbox modelled in KISSsys.

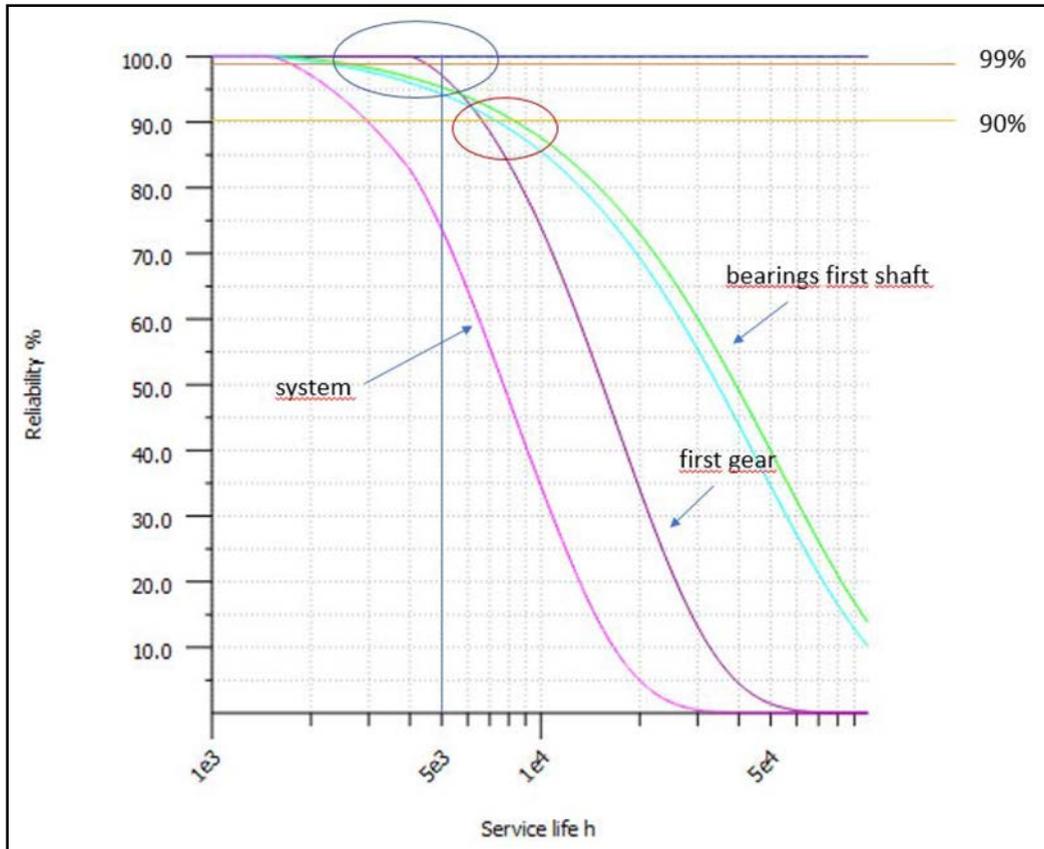


Figure 5 Calculated reliability curves.

hope that further tests could narrow this range down.

Equation (1) for  $R(t)$  can now be used to display the progression of reliability over time (or number of cycles) as a graphic. The load cycle values  $t_0$  and  $T$  can then be calculated after a service life calculation. Equations 4–6, using the achievable service life  $L_p$ , can be used for this purpose.

**An example application.** To illustrate the differences between the concept of safety factors and reliability, an example is shown. Figure 4 shows the model of a two-stage gearbox in *KISSsys* (Ref. 10). The design life is 5,000h. So, the critical component appears to be gear 1 with a flank safety factor of slightly below 1.0 (0.996) (see box “1”). The bearings have a calculated lifetime of above 7,000 hours (see box 2) and thus seem to be on the safe side.

However, looking at the graph in Figure 5 showing the reliability curves of some selected elements of the gearbox, the situation is more complex. In this graph the reliability was calculated according to the method of Bertsche (Ref. 2). The dark red curve shows the reliability of the flank for gear 1. For most probability levels, this curve is left of the two bearing curves shown, confirming the assessment from before. But this is only true for relatively low probabilities; the lower horizontal orange line is on the 90% probability level. Here, gear 1 indeed has the shorter lifetime. Still, this is above the required 5,000h design life, which is marked with the vertical blue line.

At 99% probability, the bearing life is much lower — about 3,000 hours. This is marked with the upper horizontal dark orange line. So, the more critical components are indeed the bearings. The curve for gear 1 crosses the 99% line left of the 5,000-hour bar, confirming the safety factor smaller than one.

Finally, the pink curve shows the reliability of the whole system. It is, by definition, always the left-most curve. The more components are considered, the further left this curve is moving. It is interesting to observe that for the system, the lifetime with a probability of 99% survival is about 1,800 hours, and for 90% reliability, it is about 3,000 hours. The probability of reaching the design life of 5,000 hours with this gearbox without failure is only about 73%.

## Comparison of Bertsche with the Standards

Some standards, such as AGMA 2001-D04 (Ref. 7) or ISO 281 (Ref. 3), foresee factors to change the underlying failure probability of the calculation. It is thus a natural question of how well these factors compare to the approach of Bertsche.

**Bearing lifetime per ISO 281.** First, we look at ISO 281. As mentioned, bearings show a wide scattering of the results when lifetime tests are conducted. Therefore the approach for bearings is slightly different than the other methods. The method does not calculate effective stresses and a safety factor for a given lifetime, but directly a lifetime that is reached with a certain likelihood. So, it is easy to compare with the formulae in Bertsche.

In ISO 281 the factor  $a_1$  is used to take different reliabilities into account (Table 3). The factor is directly multiplied to the lifetime  $L_{10}$  for 90% reliability, so it is straightforward to interpret (e.g.  $L_1$  for 99% reliability equals to  $0.25 \cdot L_{10}$ ). To compare this factor with the values used by Bertsche, we calculate  $f_{iB}$  from  $a_1$ :

Table 3 Definition of $a_1$ in ISO 281		
Reliability %	$L_{mm}$	$a_1$
90	$L_{10m}$	1
95	$L_{5m}$	0.64
96	$L_{4m}$	0.55
97	$L_{3m}$	0.47
98	$L_{2m}$	0.37
99	$L_{1m}$	0.25
99.2	$L_{0.8m}$	0.22
99.4	$L_{0.6m}$	0.19
99.6	$L_{0.4m}$	0.16
99.8	$L_{0.2m}$	0.12
99.9	$L_{0.1m}$	0.093
99.92	$L_{0.08m}$	0.087
99.94	$L_{0.06m}$	0.080
99.95	$L_{0.05m}$	0.077

Equation 3

$$R(t) = e^{-\left(\frac{t-t_0}{T-t_0}\right)^b} \quad (7)$$

can be rearranged to:

$$\ln(R(t)) = -\left(\frac{t-t_0}{T-t_0}\right)^b \quad (8)$$

With (Eq. 5), which relates  $t_0$  to the lifetime with 10% failure probability  $L_{10}$  with the factor  $f_{iB}$ ,

$$t_0 = f_{iB} \cdot L_{10} \quad (9)$$

$${}^b\sqrt{-\ln(R(t))} = \frac{t - f_{iB} \cdot L_{10}}{T - f_{iB} \cdot L_{10}} \quad (10)$$

Solving for  $f_{iB}$  gives:

$$f_{iB} = \frac{{}^b\sqrt{-\ln(R(t))} \cdot T - t}{\left({}^b\sqrt{-\ln(R(t))} - 1\right) \cdot L_{10}} \quad (11)$$

Extracting  $L_{10}$  outside of the bracket in (Eq. 4) results in:

$$T = \left( \frac{t}{L_{10}} - f_{iB} \right) \cdot L_{10} + f_{iB} \cdot L_{10} \quad (12)$$

For the lifetime  $t = L_{10}$  we have 10% failure probability, so the reliability is 90%:

$$t = L_{10} \Rightarrow R(t) = 0.9 \quad (13)$$

We set  $t = L_{10}$  in (Eq. 12):

$$T = \left( \frac{1 - f_{iB}}{{}^b\sqrt{-\ln(0.9)}} + f_{iB} \right) \cdot L_{10} \quad (14)$$

We now write the factor  $a_1 = a_1(p)$  from Table 3 dependent on the probability  $p$ .

$$R(t) = p \Rightarrow t = L_p =: a_1(p) \cdot L_{10} \quad (15)$$

Injecting (Eq. 14) and (Eq. 15) in (Eq. 10) and sorting for  $f_{iB}$  gives:

$$f_{iB} \cdot \left( \frac{{}^b\sqrt{-\ln(p)}}{{}^b\sqrt{-\ln(0.9)}} - 1 \right) = \frac{{}^b\sqrt{-\ln(p)}}{{}^b\sqrt{-\ln(0.9)}} - a_1(p) \quad (16)$$

Finally, we find a relationship between  $a_1(p)$  and  $f_{iB}$ :

$$f_{iB} = \frac{{}^b\sqrt{\frac{-\ln(p)}{-\ln(0.9)}} - a_1(p)}{{}^b\sqrt{\frac{-\ln(p)}{-\ln(0.9)}} - 1} \quad (17)$$

Transmitted power (kW/hp)	[P]	26.099 / 35.000	
Speed gear 1 (1/min)	[n]	2950.000	
Pressure angle at normal section (°)	[alfn]	20.000	
Helix angle at reference circle (°)	[beta]	14.000	
Number of teeth	[z]	27	104
Face width (mm/in)	[b]	20.32 / 0.800	20.32 / 0.800

Requirements of application	$K_R^1$
Fewer than one failure in 10,000	1.50
Fewer than one failure in 1,000	1.25
Fewer than one failure in 100	1.00
Fewer than one failure in 10	0.85 <sup>2</sup>
Fewer than one failure in 2	0.70 <sup>2,3</sup>

NOTES

<sup>1</sup>Tooth breakage is sometimes considered a greater hazard than pitting. In such cases a greater value of  $K_R$  is selected for bending.

<sup>2</sup>At this value plastic flow might occur rather than pitting.

<sup>3</sup>From test data extrapolation.

We can use this relationship to calculate the lifetime ratio  $a_{Bertsche}(p) = L_p/L_{10}$  for a given  $f_{ib}$ :

$$a_{Bertsche}(p) = \sqrt[b]{\frac{\ln(p)}{\ln(0.9)}} - f_{ib} \cdot \left( \sqrt[b]{\frac{\ln(p)}{\ln(0.9)}} - 1 \right) \quad (18)$$

Figures 6 and Figure 7 show the results. Bertsche proposes a range of  $0.1 \leq f_{ib} \leq 0.3$ . For a large range of the probability, this is fulfilled, only for reliabilities above 99.5%,  $f_{ib}$  drops below the lower limit. Figure 7 shows the lifetime ratios of ISO 281 over Bertsche. Until 99% probability, the ratio is close to 1, so both methods give nearly the same results. Then ISO 281 gets more conservative and for 99.95% reliability, the standard predicts about 50% of the lifetime compared to Bertsche.

So, for the most common range of requested reliability from 90% to 99%, both methods give similar results. For higher probabilities, ISO 281 is more conservative.

**Gear strength per AGMA 2001-D04.**

The second method we investigate is AGMA 2001-D04, “Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth” (Ref. 7). In the example given, we only focus on root fracture; the same statements can be made for flank failure (pitting).

The aforementioned method features a reliability factor  $K_R$  which reduces or increases the allowable root stress number  $s_t$ :

$$S_t = \frac{s_{at} Y_N}{S_F K_T K_R} \quad (19)$$

Table 5 shows the values of  $K_R$  for different failure probabilities. Since AGMA 2001 does not explicitly calculate an achievable lifetime (although for a given safety factor, the respective lifetime can be calculated using this standard), it is not possible, like for the bearings above, to directly compare the reliability factor with results from Bertsche. So here we use a different approach: we use a commercial software package (*KISSsoft*; Ref. 10) to calculate the achievable lifetime for a given gear set according to AGMA 2001 by varying the design life until we have a safety factor of 1.0. Then we

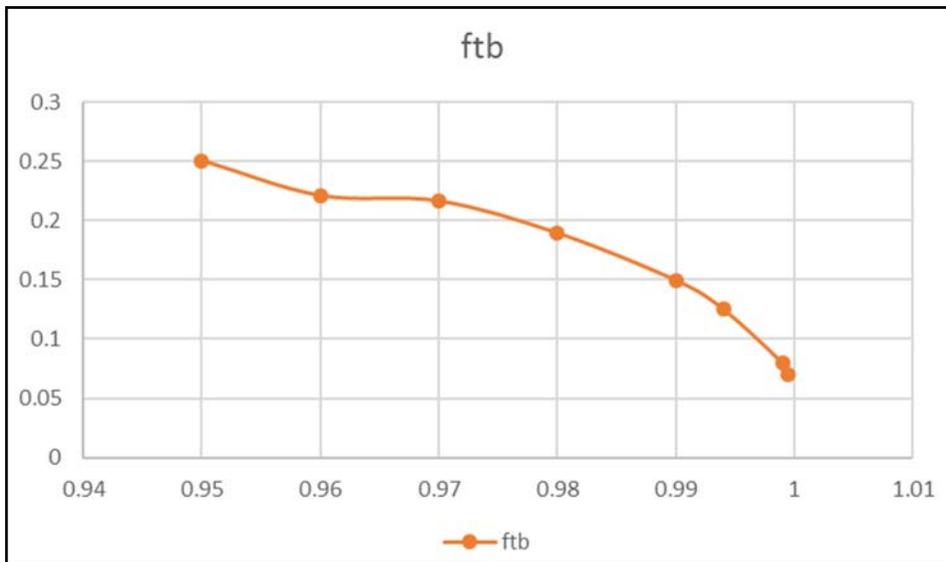


Figure 6 Calculated  $f_{ib}$  for changing reliabilities according to ISO 281.

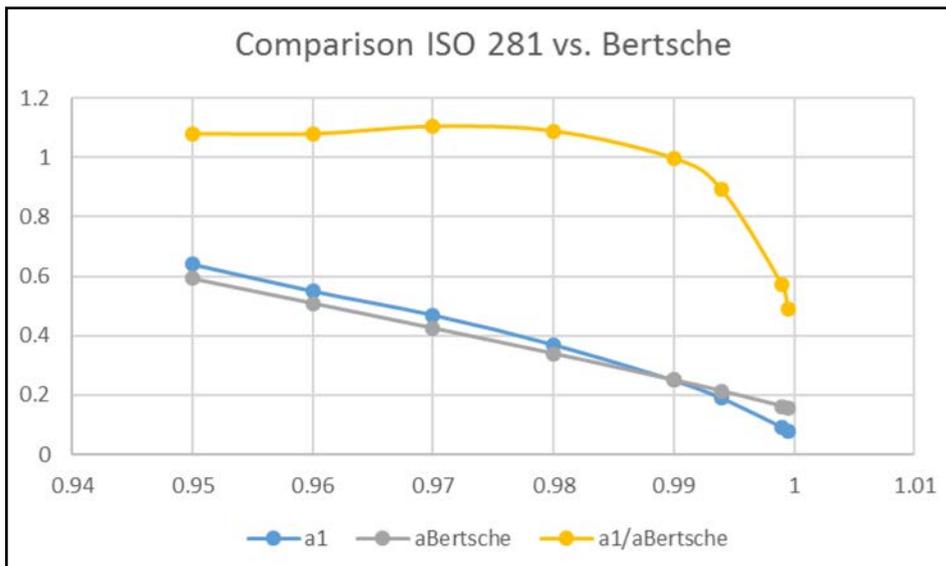
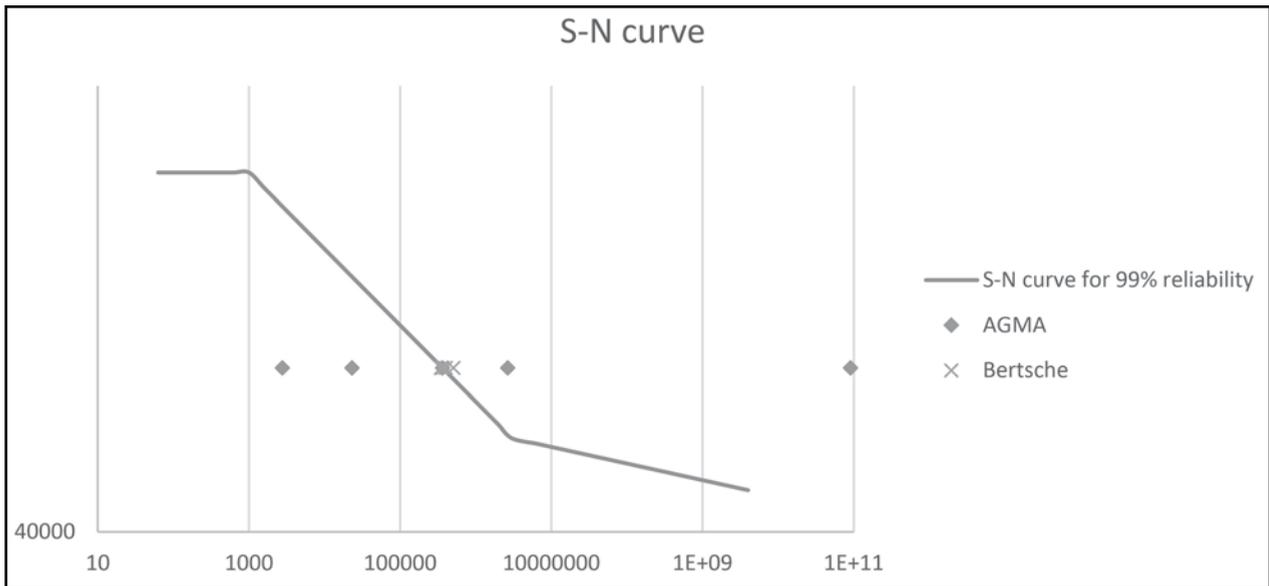


Figure 7 Comparison of  $a_1$  and  $a_{Bertsche}$



**Figure 8** Calculated lifetimes for different probabilities according to AGMA (Ref. 7) and Bertsche (Ref. 2).

use these values to calculate the Bertsche parameters  $T$  and  $t_0$ . If both methods match, the parameters are nearly constant.

The gear set is defined in Table 4.

For a reliability of 99%, we get an achievable lifetime of 7.8 hours (failure of root on gear 2). The Bertsche parameters for this point are calculated as  $T = 11.7$  hours and  $t_0 = 7.6$  hours.

Now we switch to 90% probability, which means a reliability factor  $K_R = 0.85$  is applied. The achievable lifetime increases to 57.5 hours. The Bertsche parameters are now  $T = 77.4$ h and  $t_0 = 50.3$ h. Obviously, AGMA and Bertsche use a different statistical model.

In a second experiment we calculated the lifetime for 90% reliability based on the Bertsche parameters for 99% reliability. This results in a lifetime of 8.7h. To find the corresponding value for  $K_R$ , we solve (Eq.19) for  $K_R$ :

$$K_R = \frac{s_m Y_N}{S_F K_T S_t} \quad (20)$$

We change the allowable stress number in the software manually to reach the lifetime of 8.7h. Introducing this value into (Eq. 20), we find  $K_{RBertsche} = 0.989$ . Doing the same with a reliability of 99.9%, we find  $K_{RBertsche} = 1.004$ .

Figure 8 shows the resulting lifetimes for different probability levels from 50% to 99.99%, as defined in Table 5 for both AGMA and Bertsche, plotted into the respective S-N curve for 99% reliability. While the AGMA results seem very scattered, the results from Bertsche are in a very small interval. Both results, compared to the measurements in Figure 1, seem too extreme. This, however, cannot be generalized from a single example, so further investigation in this field would make sense to optimize the statistical models of the methods.

## Summary

The reliability concept may be used to increase the transparency of the results of strength calculations of gearbox components. The method according to Bertsche is easily applicable, and the results seem reasonable. Compared to the reliability factor  $a_1$  inside ISO 281, there is a very good match of both concepts. However, in comparison to the reliability factor  $K_R$  from AGMA

2001, there are large differences. It seems that AGMA exaggerates the effect of the probability level, whereas Bertsche underestimates it. But for a final evaluation, more detailed investigations would be necessary. Looking at the improved information for the engineer coming out of the reliability concept, further work in this field seems well justified. 

## References

- Harris, T.A. *Rolling Bearing Analysis*, Edition 4, Wiley Interscience, New York, 2001.
- Bertsche, B. *Reliability in Automotive and Mechanical Engineering*, Berlin, Heidelberg: Springer Verlag, 2008.
- ISO 281:2007. Rolling Bearings — Dynamic Load Ratings and Rating Life.
- ISO 6336:2006. Parts 1–6: Calculation of load capacity of spur and helical gears.
- DIN 743:2012. Calculation of Load Capacity of Shafts and Axles.
- DIN 3990:1987. Calculation of Load Capacity of Cylindrical Gears.
- AGMA 2001-D04. Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth.
- Kissling, U. "From the Single Component Safety Factor to the System Reliability Rating," *MPT 2017*, Kyoto, JSME.
- Bronstein and Semendjajew. *Handbook of Mathematics*, 6th Edition, Harri Deutsch Verlag, 2005.
- KISSsoft/KISSsys; Calculation Programs for Machine Design; [www.KISSsoft.com](http://www.KISSsoft.com).
- ISO 16269:2014. Statistical Interpretation of Data.



**Dr. rer. nat. Stefan Beermann** studied Mathematics and Computer Science in Karlsruhe. In 1996, he went to Zurich to join the gearbox company L. Kissling & Co. AG as product manager for the calculation software KISSsoft. In 1998 he switched to KISSsoft AG as one of its first employees. Today he is CEO of KISSsoft AG, together with the company's founder Dr. Ulrich Kissling.

