Efficient Layout Process of Cylindrical Gears with Manufacturing Constraints

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Introduction

Cylindrical gear design can be divided into three steps. In the first step, rough gear pair dimensions such as center distance and face width are being estimated. Center distance and face width are directly linked to the available space (housing dimensions) and influence the overall size, weight and cost of the gears. In addition, the torque capacity strongly depends on the chosen gear materials, heat treatment and gear quality. Although case-hardened gears tend to give a higher torque capacity than nitrided gears, a final machining process like for example grinding is required to compensate the hardening distortion. Considering all these factors in a gear rough sizing and finding the best solution becomes a tough challenge.

In the next step, gear macro geometry is defined. In a conventional gear manufacturing process, the choice of normal module, pressure angle and reference profile are directly linked to the cutter geometry. The consideration of available tools in the early design stage can save a lot of effort in the later manufacturing steps. In addition, the resulting gear geometry must satisfy the required safety factors in accordance with the selected gear strength calculation method. Although a higher gear root radius tends to give higher root safeties, it may produce contact interference and require a special cutter. Evaluating different geometric solutions and eliminating non-feasible ones in the early design phase becomes an important task.

In the last phase, gear micro-geometry is defined. The aim of this step is to specify flank line and profile modifications for optimal contact pattern, lower noise emissions and various other parameters. Here, the choice of modification parameters is directly linked to the final machining process. Often a grinding worm with associated dressing wheel is used. If a specific list of grinders/dressers is available, it makes sense to consider them in the layout process to avoid extra costs in the manufacturing process.

In this paper we will present an efficient gear layout procedure based on international standards for gear geometry and strength calculation with the consideration of manufacturing constraints such as lists of hobs, grinders and dressers. The aim is to reduce costs in the later manufacturing steps or alternatively, to be able to predict the need for additional tools in the early design process.

In the Gear Rough Dimensioning section, an approach for gear rough dimensioning is presented. The resulting center distance and face width are used for the next step, i.e.—gear fine-sizing. In Gear Macrogeometry and Optimization, the focus is on gear macro geometry. Parameters such as normal module, pressure angle, helix angle and reference profile are found to meet certain optimization criteria. Two macro-geometric solution examples are used for further analysis; this is followed by Gear MicroGeometry and Optimization. Lead modifications are used to optimize the load distribution along the flank line. Profile modifications and contact analysis are used to optimize the load distribution in profile direction and reduce gear noise. Each chapter also focuses on selected manufacturing constraints that influence the overall design process of cylindrical gears.

Gear Rough Dimensioning

In the first step, gear rough dimensioning is performed. The aim of this step is to find the optimal center distance and face width. These two parameters basically define the overall gearbox dimensions—either for a one-stage or a multi-stage gearbox—and are directly linked to the housing dimensions. The estimation is based on the selected calculation method for gear geometry and strength. The calculation method is defined by the chosen standards (DIN, ISO, AGMA, VDI, GOST, etc.), and the required safety factors; typically for root

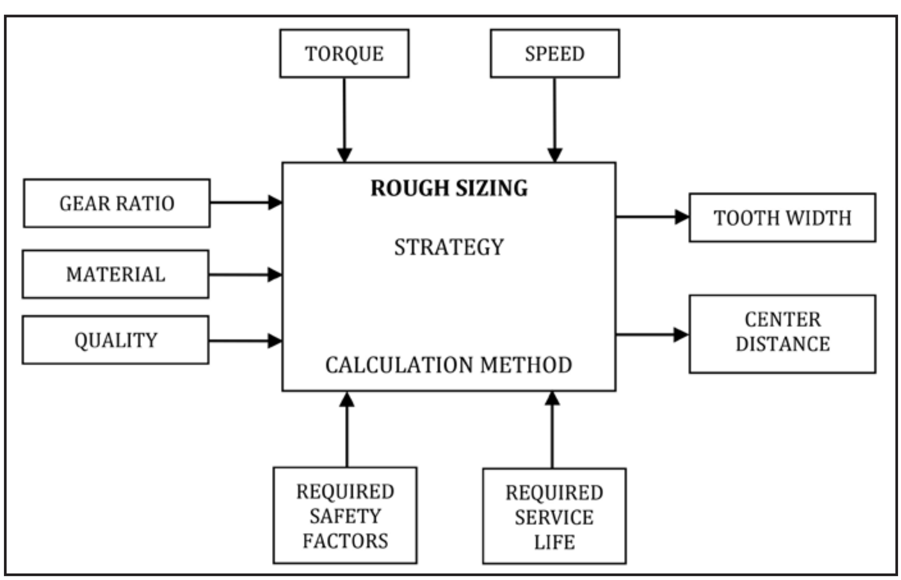

Figure 1 Rough sizing procedure.

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a [mm]	b_1 [mm]	b_2 [mm]	m_n [mm]	z_1	z_2	i	$SF_{smallest}$	$SH_{smallest}$	P_{max} [kW]	W [kg]	T_{max}/W [Nm/kg]	dB(A)
97.660	60.930	59.440	2.972	16	49	3.063	2.330	1.000	49.952	8.547	37.205	77.413
100.000	61.232	59.482	3.000	16	50	3.125	2.322	1.000	49.970	9.026	35.246	77.997
106.000	49.540	48.040	2.500	21	62	2.952	1.692	1.002	50.176	8.139	39.247	78.171
106.000	50.497	48.997	2.750	19	56	2.947	1.871	1.001	50.059	8.282	38.477	78.382
106.000	50.408	48.908	2.500	21	64	3.048	1.801	1.001	50.069	8.334	38.246	77.533
106.000	51.000	49.500	2.750	19	58	3.053	1.958	1.001	50.107	8.417	37.897	77.659
106.000	52.974	51.474	3.000	17	52	3.059	2.135	0.998	49.786	8.763	36.167	78.341
106.000	52.951	51.451	3.000	17	53	3.118	2.174	0.998	49.790	8.785	36.083	77.960
106.000	54.602	53.102	3.000	17	54	3.176	2.242	1.000	49.962	9.090	34.992	77.691
106.000	55.972	54.472	2.750	18	57	3.167	1.954	0.999	49.944	9.394	33.846	78.568
106.000	55.997	54.497	2.750	19	59	3.105	2.282	0.994	49.445	9.367	33.605	77.251
106.000	61.196	59.696	3.000	16	52	3.250	2.309	0.999	49.894	10.359	30.662	79.068
111.292	43.990	42.920	2.146	25	78	3.120	1.492	1.000	49.987	8.162	38.987	77.012
112.000	42.797	41.484	2.250	25	74	2.960	1.471	0.999	49.925	7.868	40.396	77.661
112.000	43.813	42.500	3.000	19	56	2.947	2.057	0.999	49.945	7.975	39.869	77.469
112.000	44.781	43.468	2.750	20	61	3.050	1.832	1.000	49.998	8.270	38.488	77.761
112.000	45.130	43.817	2.250	24	74	3.083	1.456	0.998	49.827	8.416	37.693	78.094
112.000	45.307	43.994	2.250	24	75	3.125	1.509	0.998	49.805	8.472	37.424	77.783
112.000	46.554	45.241	3.000	18	55	3.056	2.012	1.000	49.969	8.610	36.949	78.304
112.000	46.485	45.172	2.750	20	62	3.100	1.916	0.998	49.794	8.617	36.787	77.521
112.000	47.845	46.532	3.000	18	57	3.167	2.129	1.000	49.955	8.904	35.716	77.668
112.000	48.688	47.375	2.500	23	68	2.957	2.025	0.999	49.903	9.027	35.193	76.962
112.000	48.116	46.803	2.500	21	67	3.190	1.663	0.999	49.946	9.056	35.113	78.253
112.000	48.239	46.926	2.500	21	68	3.238	1.713	0.999	49.946	9.104	34.927	77.924
112.000	49.552	48.239	2.750	19	60	3.158	1.846	0.999	49.932	9.298	34.186	78.712
112.000	52.968	51.655	3.000	17	55	3.235	2.150	0.999	49.899	10.008	31.741	78.996
118.000	41.232	39.919	2.250	25	79	3.160	1.423	0.999	49.928	8.575	37.065	77.918
118.000	41.764	40.451	2.250	25	78	3.120	1.383	0.999	49.400	8.666	36.288	78.256
118.000	41.782	40.469	2.250	25	80	3.200	1.474	0.993	49.330	8.715	36.033	77.669
118.000	44.976	43.663	2.250	24	78	3.250	1.443	0.999	49.881	9.472	33.524	78.792
118.000	45.662	44.349	2.250	25	81	3.240	1.744	1.001	50.105	9.632	33.115	77.328
141.955	28.080	25.920	4.320	16	49	3.063	2.266	1.000	50.009	7.923	40.184	77.413
161.648	20.260	18.700	3.117	25	78	3.120	1.447	1.000	50.014	7.545	42.201	77.012

Figure 2 Rough sizing: a — center distance; b — face width; mn — module; z — number of teeth; i — gear ratio; SF — root safety; SH — flank safety; P — max transmittable power; W — weight, Tmax/W — power density.

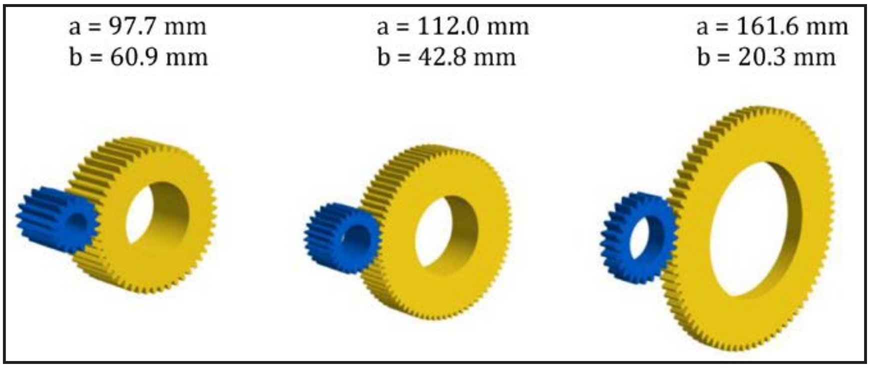

Figure 3 Solution with small center distance vs. high center distance.

and flank, but in some cases also for scuffing, micro-pitting, wear etc.

The main input parameters for rough sizing are torque (or power), speed, required gear ratio, gear material and quality, heat treatment and requested service life (Fig. 1). According to ISO 6336 (Ref. 1) material endurance limits depend on the surface hardness, material quality and heat treatment. Consequently, if the gearbox dimension requirements are not met, a different material, heat treatment or a higher gear quality should be applied.

Once the input data, required safety factors, and the calculation method are fixed, a batch calculation can be performed to extract different geometric solutions. The idea is to search for all feasible solutions in terms of center distance and face width while the safety factors are met and a reasonable quotient between face width and normal module b/mn is maintained (for automotive applications typically b/mn ≈ 6, for industrial gearboxes b/mn ≈ 20). Figure 2 shows an example of such a batch calculation, sorted in terms of center distance. In this early stage, it is possible to estimate the size, weight and power density range of the gearbox variants.

The solution with the smallest center distance results in small gearbox dimensions but is less attractive in terms of weight or power density. The solution with the highest center distance has, in this example, lowest weight and highest power density but is due to upper limits for gearbox size not applicable. The solution in between (a = 112) gives a good trade-off between small gearbox size, low weight and second highest power density.

Figure 3 illustrates two solutions with minimum and maximum center distance. Both solutions are feasible, since the requested safety factors are met. At this point the engineer must make a choice, which solution is optimal for the given application. Different strategies can be applied during the search for an optimum solution.

An often-used approach is to minimize the weight of the gears due to its direct link to manufacturing costs (material price per kg). Figure 2 illustrates the optimization potential for a 50 kw gearbox: the choice of the center distance may save up 25% of gear weight (solutions between 5.4 kg and 7.1 kg). A further approach would be to maximize the power density of the gearing (max. transmittable torque / per kg) — a nice trade-off between low weight and high transmittable power. In some applications e.g. plastic gears, the solutions with minimum center distance (smallest gearbox dimensions) are preferred, due to limited space in the end-product, or with the highest module to reduce the influence of manufacturing tolerances.

Thus, gear rough sizing is an important step in the gear design process. It provides a range of possible gearbox dimensions and allows performing an optimization of gear weight, size and manufacturing costs. In the next step, gear macro geometry needs to be optimized.

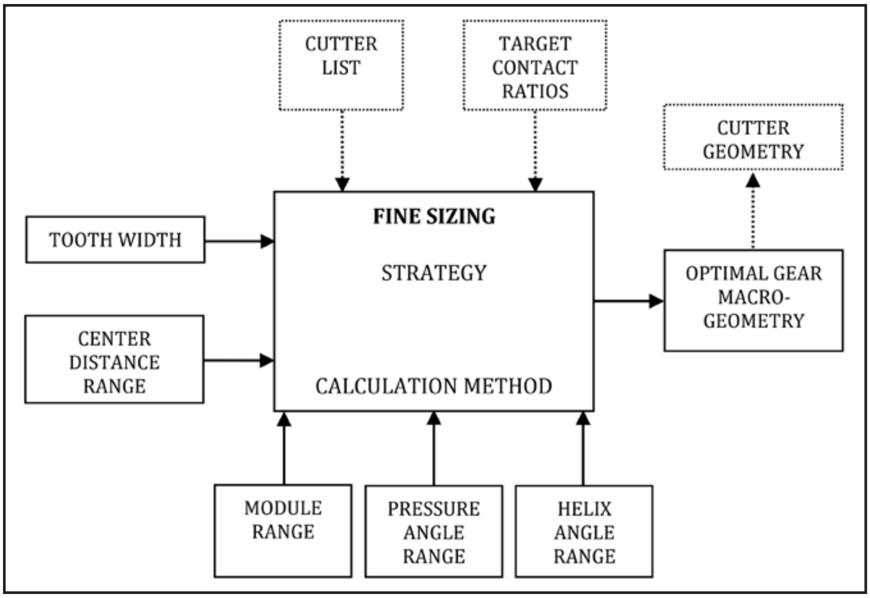

Figure 4 Fine-sizing.

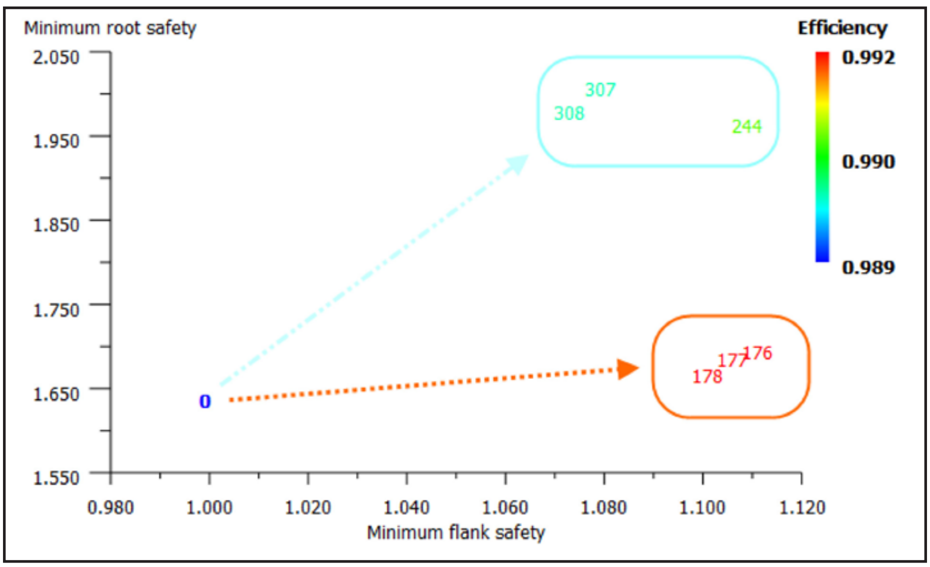

Figure 5 Fine-sizing and analysis in terms of safety.

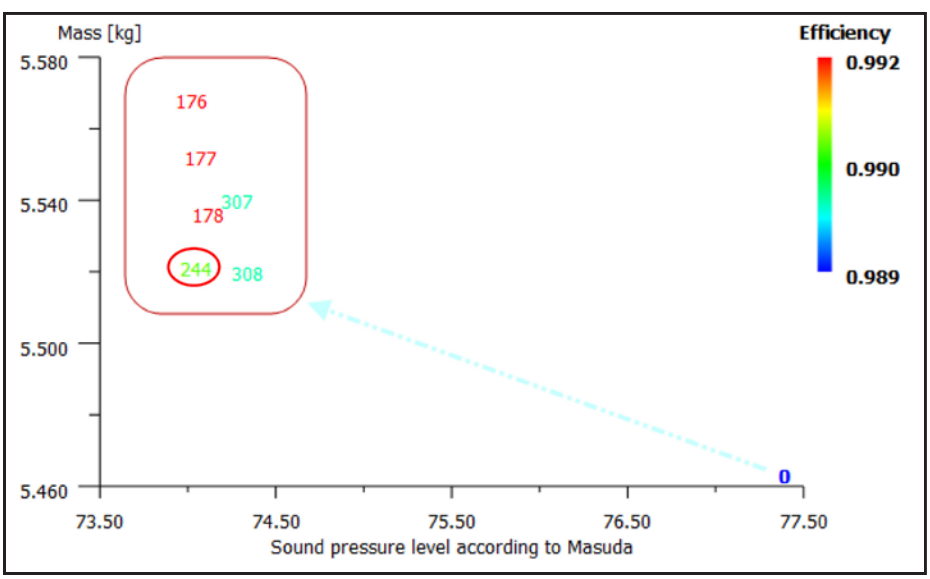

Figure 6 Fine-sizing, analysis in terms of sound pressure level, mass and efficiency.

Gear Macrogeometry and Optimization

Once the center distance and face width are fixed, gear macro geometry can be optimized with the so called fine sizing strategy. The main idea is to perform a variational calculus of gear main parameters, such as normal module, pressure angle, helix angle, number of teeth, profile shift and reference profile (Fig. 4). With modern software tools, such a calculation may easily produce over 1,000 different geometric solutions. The main challenge is to eliminate all non-feasible variants and apply a clever strategy to find an optimum.

For conventional hobbing or pinion type cutting, the choice of normal module, pressure angle and gear reference profile is directly linked to the tool geometry. Thus, manufacturing constraints may limit the number of feasible solutions. Figure 4 shows two possible approaches: a list of available cutters may be used as input to the fine sizing procedure to save costs that may arise due to the need of a special cutter in the later manufacturing process. On the other hand, if the optimized gear design shall be unique, the cutter geometry becomes a result of the calculation and opens new optimization potential. For example, one may define a desired transverse contact ratio ($eps\_a \geq 2$) and use this constraint to iterate over the gear reference profile to get solutions with high contact and lower variation of the contact stiffness.

While keeping the center distance and face width fixed, one can now iterate over a range of normal module, pressure, and helix angle over different combinations of number of teeth and profile shift coefficients while eliminating solutions:

- where minimum safety factor requirements are not met
- with an undercut
- where tooth thickness at the tip is too small
- where deviation from the requested gear ratio is too large
- where specific sliding (wear, friction) is too high

A simple algorithm can sort and extract best solutions in terms of gear ratio, strength, weight, contact stiffness, specific sliding etc. and provide a list of best overall solutions, based on a weighted combination of the above criteria. Figures 5 and 6 illustrate the final stage of such an optimization, where only several best solution candidates remain. Solution Nr. 0 was the starting point after the rough sizing procedure. All other solutions are the result of the optimization, while maintaining the same center distance and face width (same gearbox dimensions).

All optimized solutions show a significant increase in safety factors, efficiency and a lower sound pressure level according to Masuda (Ref. 5). Solution number 244 shows a good trade-off between high strength and efficiency with low mass and noise level.

Thus, we have introduced one effective method to optimize gear macro-geometry under given manufacturing constraints. In the next step, gear micro-geometry is defined. For further analysis, we will use two different gear variants (Fig. 7) from the fine sizing procedure. The gearing on the left has a standard ISO 53: reference profile C (Ref. 2). The gearing on the right has a deep tooth profile, optimized for a transverse contact ratio of 2 and has a slightly lower normal module. Other parameters like center distance, face width, pressure angle, helix angle and gear ratio remain the same.

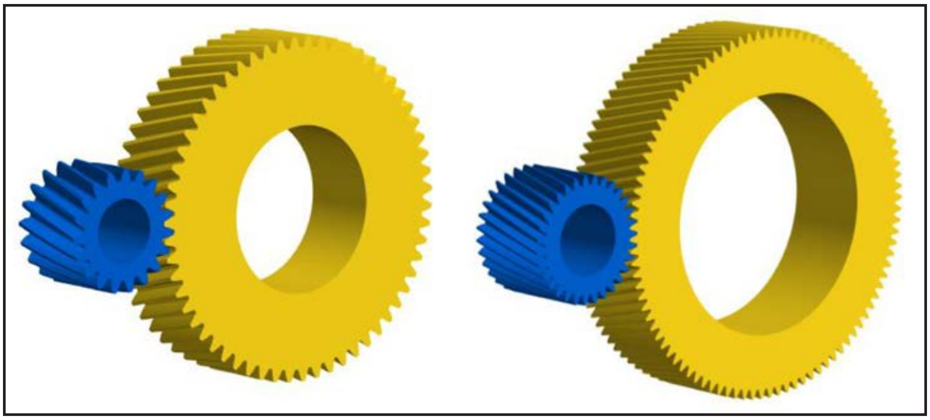

Figure 7 Example of two optimal geometric solutions with standard reference profile (left) and deep tooth profile (right).

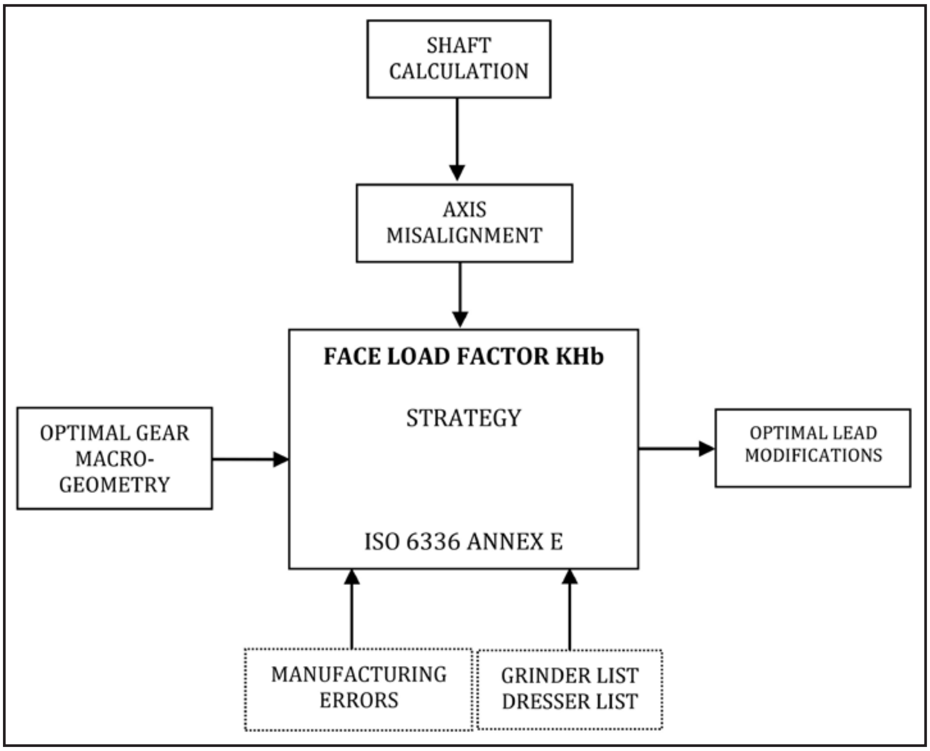

Figure 8 Optimization of face load factor with lead modifications.

Gear Microgeometry and Optimization

In the third and final step, gear micro-geometry is defined. First, flankline modifications are applied to compensate shaft bending and torsion, misalignments due to manufacturing errors, bearing clearance, deformation and influence of the housing.

Optimal flankline modifications will normally increase the torque capacity of the gearbox due to a more even load distribution along the flank, thus reducing the face load factor $K_{H\beta}$. Typically, a helix angle modification is applied to compensate shaft misalignments, and a crowning to compensate the random manufacturing errors and torsional effects.

Once the load distribution along the flank is optimal, profile modifications are applied to reduce gear noise. Other effects like lower contact temperature and higher efficiency, smooth normal force distribution or higher micropitting resistance may be achieved. However, in this paper we will focus on the optimization of noise related parameters such as the contact path under load, peak-to-peak transmission error, force excitation and harmonics.

Flankline (lead) modifications. The face load factor $K_{H\beta}$ is defined as the ratio between the highest line load divided by the average load over the face width (Ref. 1). Thus, under optimal conditions the face load factor would be equal to one. ISO 6336 Annex E describes one possible approach to calculate the face load factor $K_{H\beta}$, while considering shaft misalignment due to bending, torsional deformation and manufacturing errors (Fig. 8). Flankline modifications are applied to compensate the uneven load distribution.

For gears with higher quality, the face load factor mainly depends on the shaft deformation due to bending. It is thus important to perform a shaft deformation calculation when optimizing the load distribution. Figure 9 illustrates the results of an analytical calculation of shaft deformation with consideration of non-linear bearing stiffness resulting from bearing inner geometry,

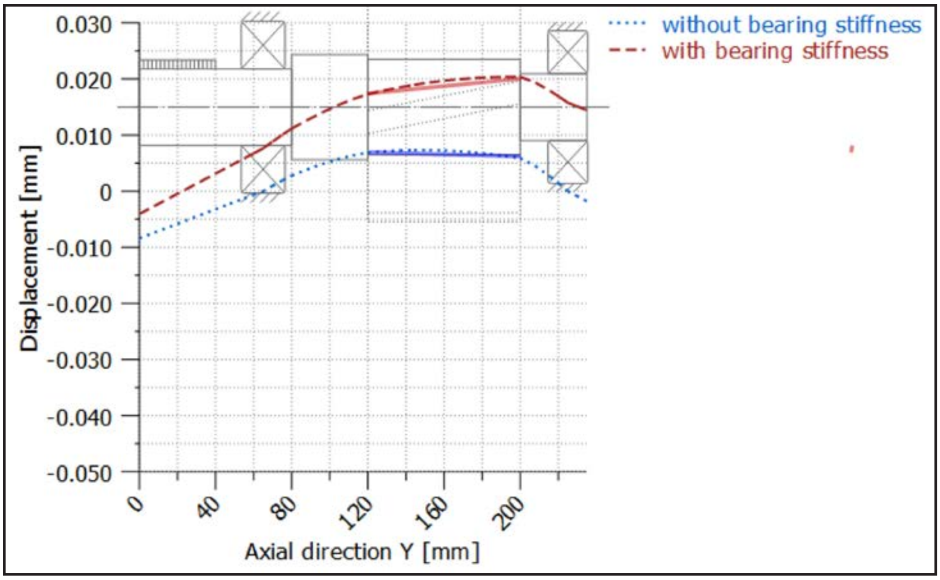

Figure 9 Shaft bending line.

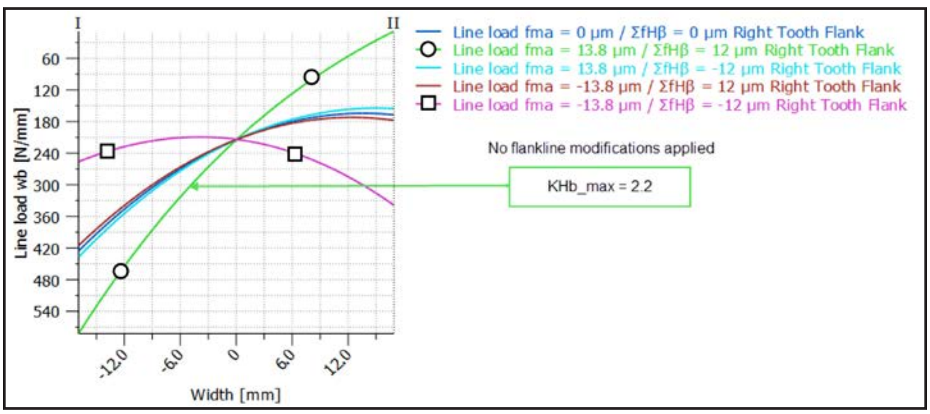

Figure 10 Resulting line load without flankline modifications.

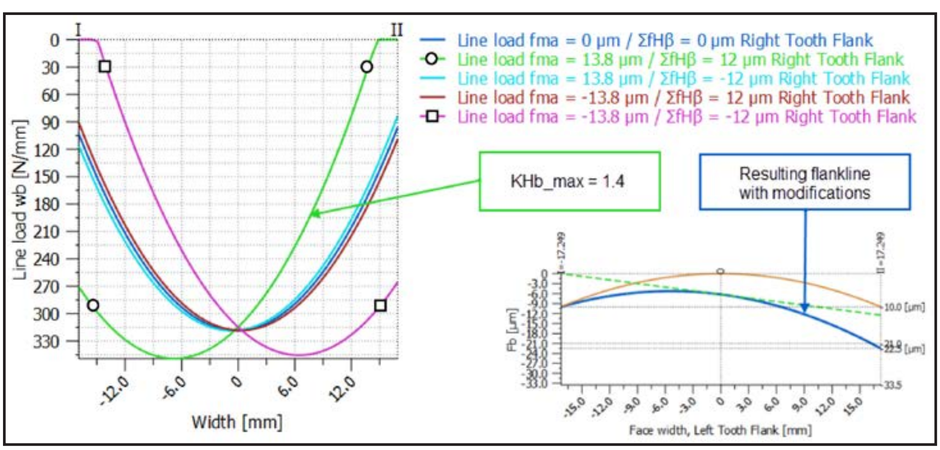

Figure 11 Resulting line load with flankline modifications.

as described in ISO/TS 16281 (Ref. 3) (see red curve). The blue curve illustrates the bending line with infinite bearing stiffness. Thus, if bearing stiffness is not considered, the sizing of a helix angle modification can be wrong and, in some cases, even provide worse load distribution than if no modification applied at all.

In addition, manufacturing allowances like axis non-parallelism (f_{ma}) and helix slope deviation ($f_{H\beta}$) should be considered. Since manufacturing errors can either have a positive or a negative sign, several scenarios must be analyzed. Figure 10 illustrates the load distribution for 5 different cases:

- $f_{ma}=0$ and $f_{H\beta}=0$
- f_{ma} (+) and $f_{H\beta}$ (+)
- f_{ma} (+) and $f_{H\beta}$ (-)
- f_{ma} (-) and $f_{H\beta}$ (+)
- f_{ma} (-) and $f_{H\beta}$ (-)

According to ISO 6336, Annex E, the strength calculation has to be performed with the highest face load factor (in this example, $K_{H\beta}=2.2$) that results from the above 5 cases.

To optimize the load distribution and the face load factor with manufacturing errors, an additional crowning is typically applied. The idea is to reduce the maximum line load and for all scenarios of manufacturing errors and shift the peak line load away from the gear edges. Figure 11 illustrates the line load with an additional crowning of 10 microns. In this example, the highest face load factor appears for f_{ma} (+) and $f_{H\beta}$ (+) and equals around 1.4. Applying the resulting face load factor in the strength calculation resulted in 45% higher root safety and 20% higher flank safety.

One important manufacturing constraint to consider here is the manufacturing twist. When using generation grinding to produce crowning in helical gears, a twist may appear due to the grinding motion of the tool. If not compensated, this may lead to a higher effective line load. Figure 12 illustrates the shape of the crowning with the influence of the twist due to manufacturing. The compensation of the twist is not a simple task — especially due to the lack of available literature — and requires the use of modern grinding machines.

Loaded tooth contact analysis. The aim of loaded tooth contact analysis (LTCA) is to evaluate the gear mesh under load. For the calculation of tooth deformation, a tooth stiffness model is required. An analytical model for tooth deformation was presented by Weber and Banaschek (Ref. 6), where gear deformation is divided into three main components:

- Gear body deformation
- Tooth bending deformation
- Hertzian flattening

Based on this theory, an analytical stiffness model can be created. A loaded tooth contact analysis can then be performed based on the tooth deformation, shaft misalignments, manufacturing errors (e.g. pitch error), and a defined partial load for the calculation (Fig. 13). The results of LTCA provide important parameters for noise characterization and optimization:

- Transmission error
- Amplitude spectrum of the transmission error
- Force excitation
- Path of contact under load

The transmission error (TE) describes the deviation of the theoretical contact point from the point of contact under consideration of tooth deformation. Especially the peak-to-peak transmission error (PPTE) is a valuable parameter for noise optimization. The Fourier transformation provides the orders of harmonics and allows evaluation of the excitation frequencies. From the transmission error and contact stiffness, it is possible to derive the excitation force (EF) (Ref. 7) that allows comparison of different geometric solutions in terms of vibration excitation and—along with the transmission error—to find the best variant with reduced gear noise. Furthermore, the path of contact under load shows the change of pressure angle at the beginning and end of the meshing. This phenomenon will later be referred to as "contact shock" (Fig. 14).

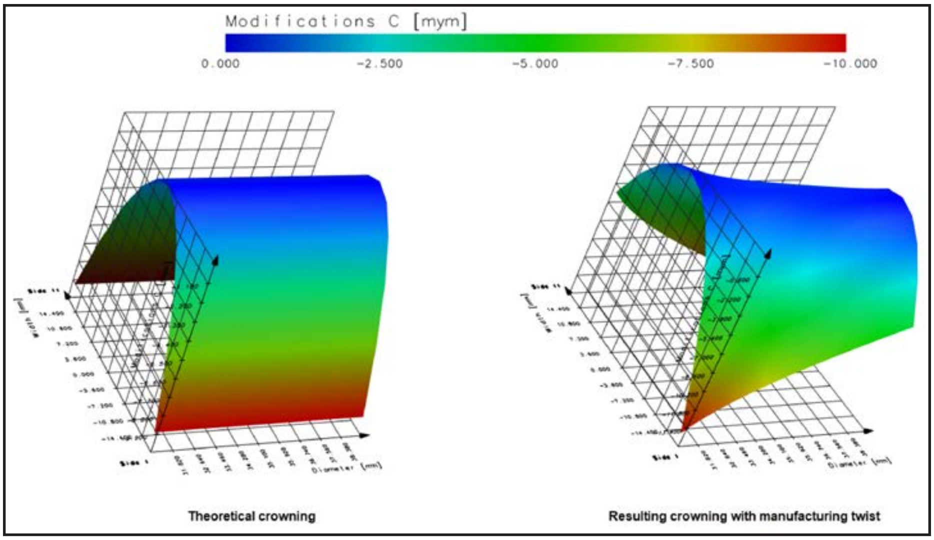

Figure 12 Influence of manufacturing twist on crowning.

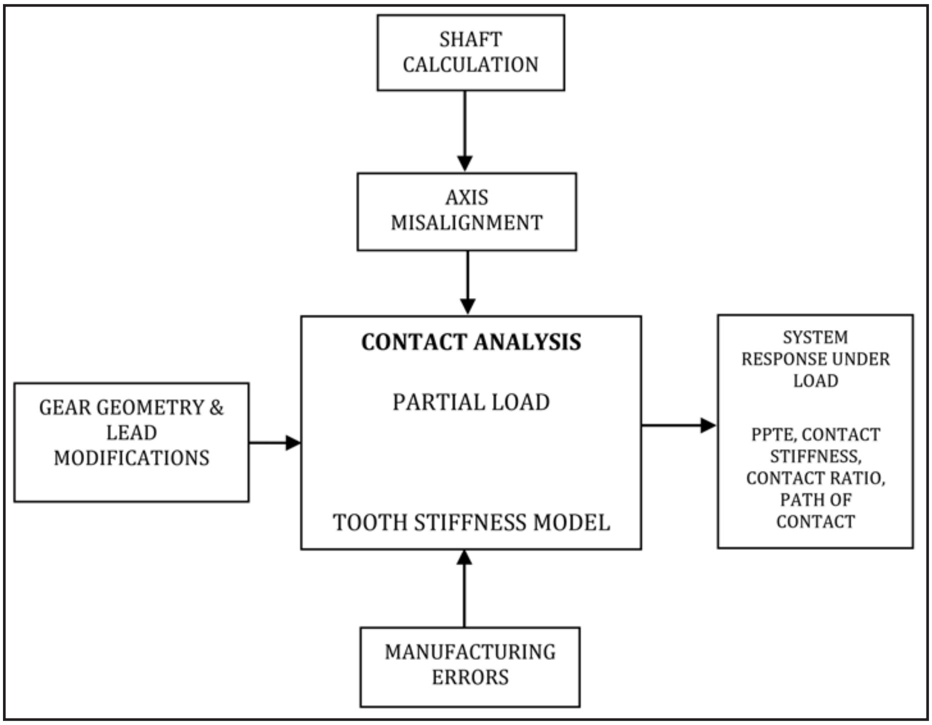

Figure 13 Contact analysis.

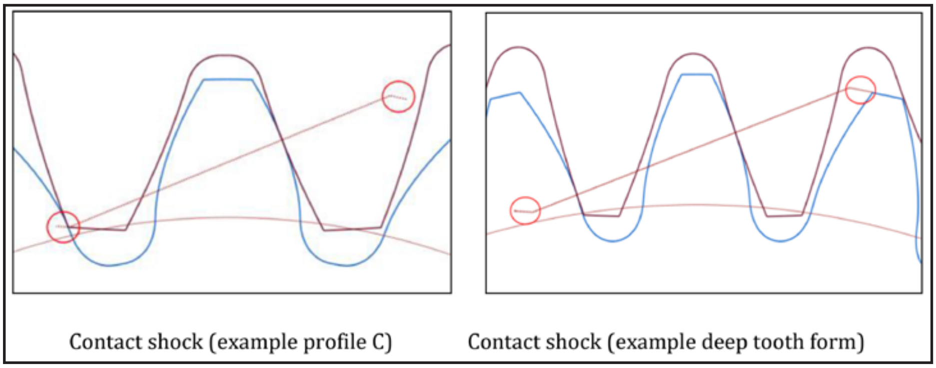

Figure 14 Contact shock.

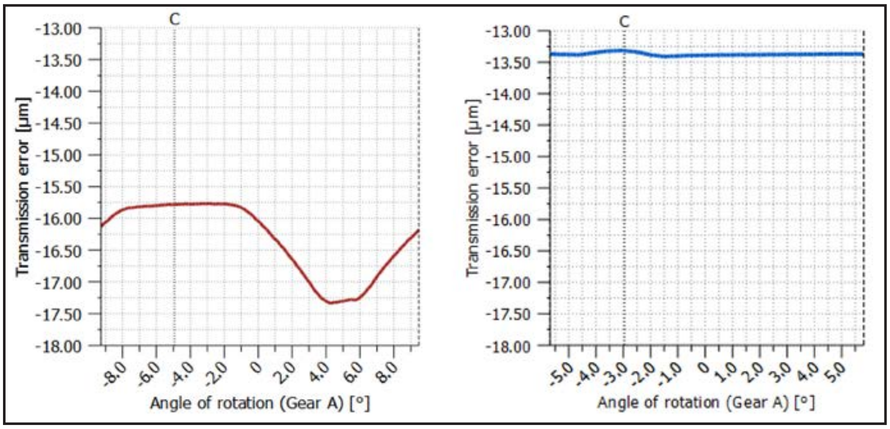

Figure 15 Transmission error.

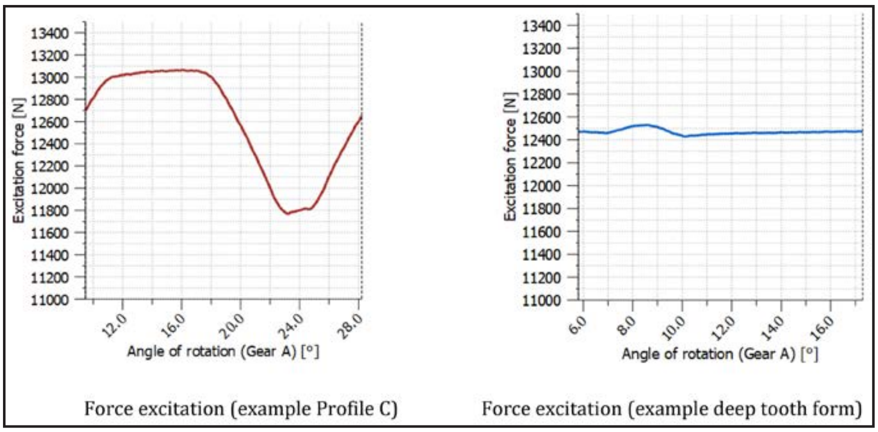

Figure 16 Excitation force.

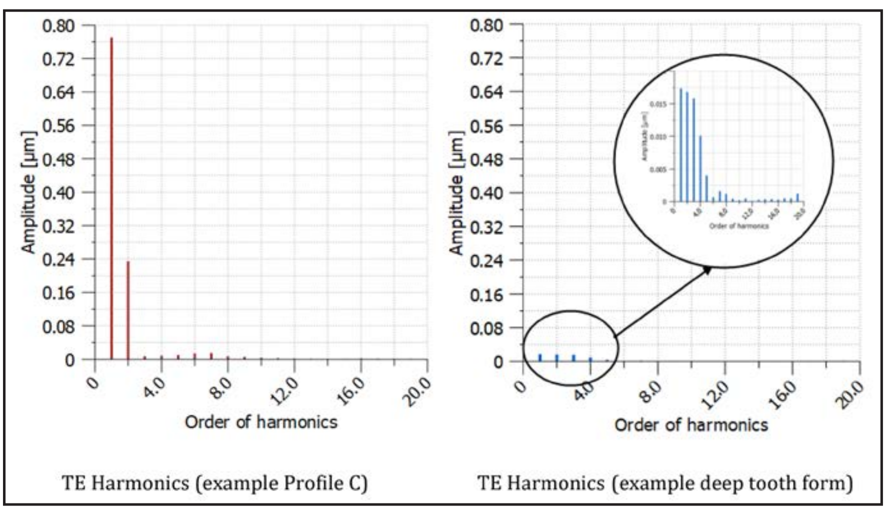

Figure 17 Amplitude spectrum.

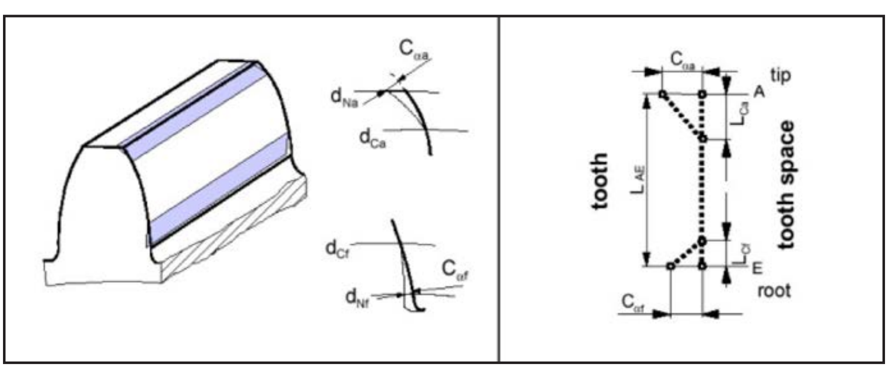

Figure 18 Tip and root relief.

In the previous Gear Macrogeometry and Optimization section, two example solutions from the fine-sizing procedure were presented. Figure 17 illustrates the results of a loaded tooth contact analysis. Both solutions show a contact shock in the beginning and at the end of the mesh. Due to a higher contact ratio and a higher stiffness, the amplitudes of transmission error and force excitation are lower for the solution with a deep tooth form. On the other hand, the amplitude spectrum shows less significant order of harmonics for the standard reference profile.

Profile modifications. The final step is the sizing of profile modifications. Different features such as noise, contact temperature, efficiency, micro-pitting or scuffing can be improved with well sized profile modifications. In this paper we will focus on the reduction of noise with following a simple strategy:

1. Eliminate contact shocks at the beginning and at the end of the mesh.
2. Reduce the amplitude of the transmission error (PPTE).
3. Reduce the second and higher order of harmonics to become as close as possible to zero.

In the ISO 21771 different modification types are defined. Typically, a tip relief (Fig. 18) on both gears is applied to reduce gear noise. The amount of tip relief $C\alpha a$ is adjusted to eliminate contact shocks and the tip relief roll length Lca is chosen to minimize PPTE.

Figure 19 illustrates the path of contact under load with eliminated contact shocks when applying a tip relief of $C\alpha a = 22\,\mu m$ in the example with a standard reference profile and $C\alpha a = 15\,\mu m$ for the deep tooth form example.

The peak-to-peak transmission error (Fig. 20) was reduced in both cases. The lowest noise levels can be expected for the example with the deep tooth form: the transmission error curve is smoother and has a lower amplitude. This can be explained with the higher contact ratio and the nearly constant stiffness of the deep tooth form. Same effects can be observed for the excitation force (Fig. 21). Although the average value of the excitation force did not change for both variants, the amplitude levels are decreased. Figure 22 illustrates another advantage when applying a tip relief, i.e. — the second and higher order of harmonics were reduced, especially for the example with the deep tooth form. The shape of a signal with only one dominant harmonic is similar to a sine/cosine wave. On the contrary, if many orders of harmonics are dominant, the signal has rather the shape of a square function, which is unfavorable.

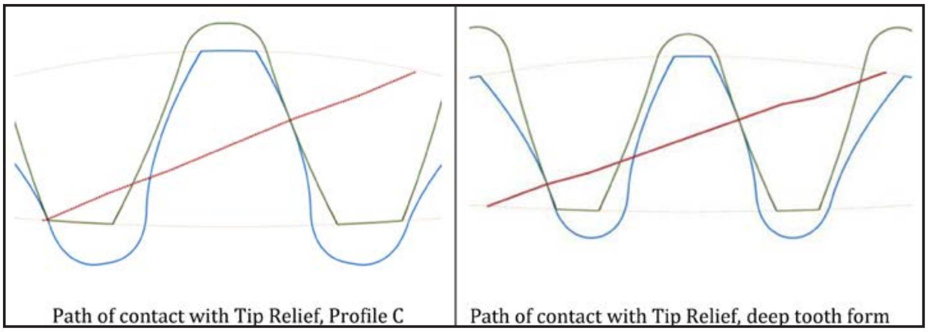

Figure 19 Optimized contact path.

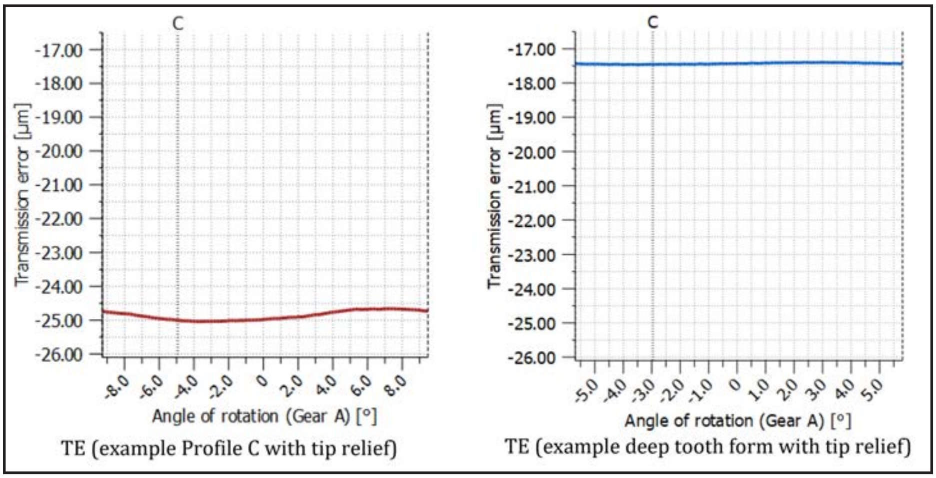

Figure 20 Optimized transmission error.

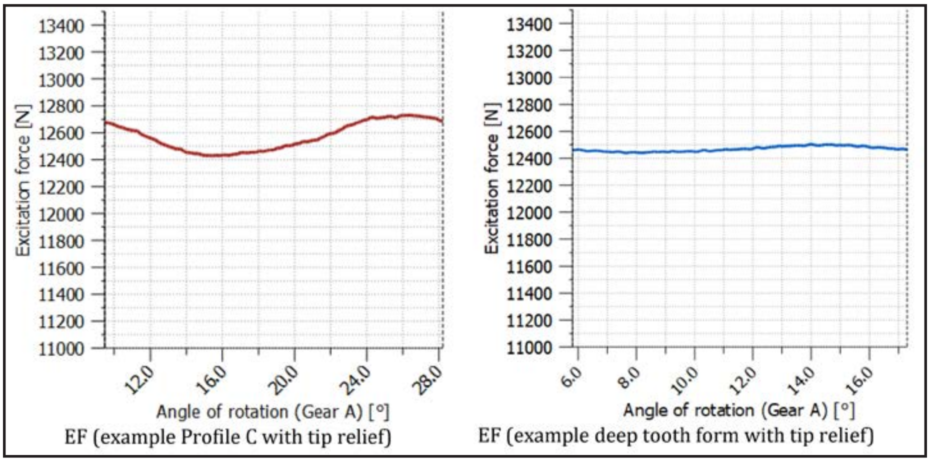

Figure 21 Optimized force excitation.

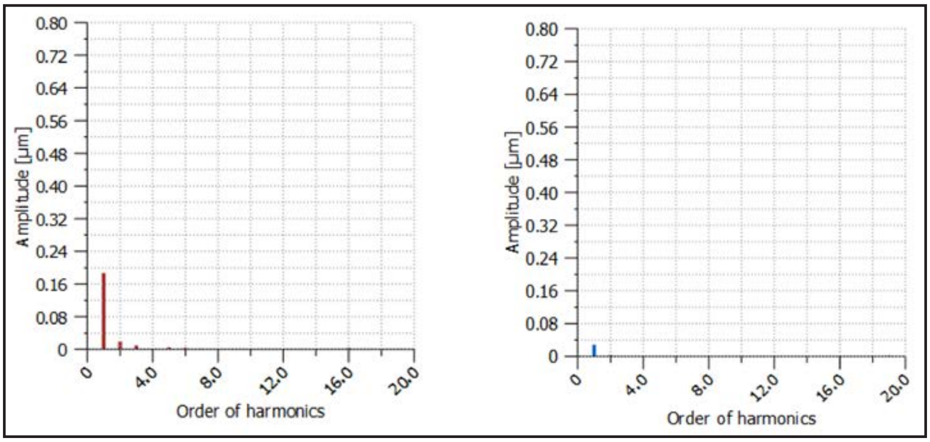

Figure 22 Optimized amplitude spectrum.

Figure 23 illustrates the profile diagrams of both variants. If manufactured with generation grinding, a dressing wheel is typically used to trim the grinding worm that later produces the modifications. The evaluation, if a specific modification can be carried out with available grinders/dresses can save costs and time. In some cases, modifications are directly integrated into the shape of the hobbing cutter.

Gears are usually manufactured with a grinding stock that will be removed in the grinding process to meet dimensional requirements. The choice of the gear heat treatment method basically defines the amount of grinding stock for the finishing process (Ref. 8). By selecting a heat treatment processes with smaller distortion, the amount of grind stock can be reduced to minimize machining of hardened surfaces and reduce the overall costs of manufacturing. By this, the manufacturing of modifications is linked to the heat treatment/finishing process and should be considered in the design process.

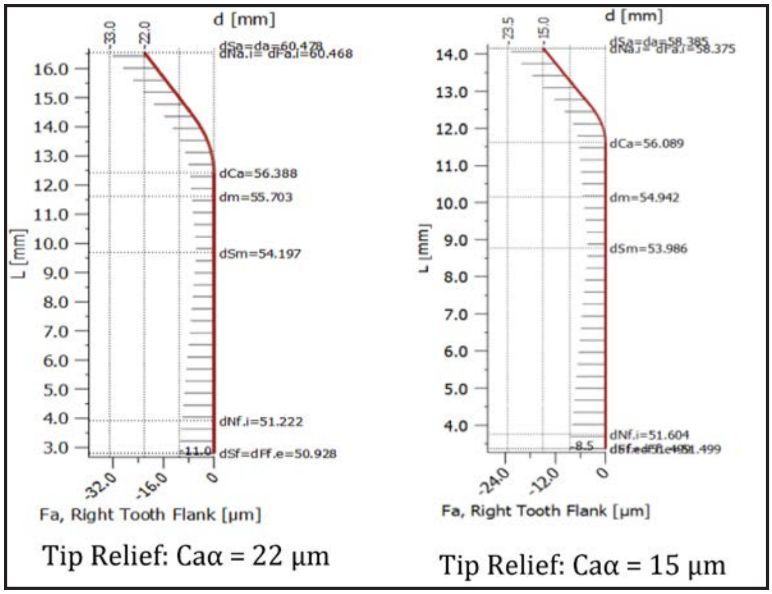

Figure 23 Profile diagrams, Profile C (left), deep tooth form (right).

Summary

The layout procedure of cylindrical gears can be divided into three main steps: rough sizing, fine sizing and modification sizing. In the rough sizing step gearbox dimensions like center distance and face width are defined. If the resulting gearbox dimensions are too large, a choice of a better material or heat treatment process can help.

In the next step, gear macro-geometry is defined. Parameters such as normal module, pressure angle, helix angle and reference profile are optimized to meet different design criteria. A list of available cutters can help to optimize the manufacturing costs. Alternatively, a non-standard reference profile can be used to achieve certain properties like, for example, a higher mesh stiffness or contact ratio.

In the final step, gear micro-geometry is defined. It is shown that applying well-designed flank line modifications can significantly increase the torque capacity of a gearbox. A manufacturing twist resulting from the manufacturing process may limit the optimization potential. A loaded tooth contact analysis allows quantification of noise parameters such as contact path under load, including contact shocks, peak-to-peak transmission error, force excitation and harmonics. Profile modifications are then applied to improve the above parameters and decrease the source of vibrations. **PTE**

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For more information.
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