BEVEL GEAR STRENGTH RATING

THE APPROPRIATE COMBINATION OF FEM WITH RATING STANDARDS

The combination of drivetrain design software and LTCA using the FEM approach allows a complementary calculation for the bevel gear rating, establishing a complete and very reliable process for the development of bevel and hypoid gearsets.

By JÜRG LANGHART and MARKUS BOLZE

Strength rating of bevel gears according to standards such as AGMA, ISO, etc. is executed based on virtual cylindrical gears, only modified by a few specific bevel gear factors. The rating method of these standards also includes the calculation of permissible stresses and finally resulting safety factors.

Furthermore, the integrated S-N curves consider also an increased permissible stress during limited life and allow a lifetime prediction.

The contact analysis for bevel gears allows a rating of the stresses. It allows the individual to consider flank modifications such as crowning, twist, etc., including the corresponding displacements. A lack of the contact analysis is the calculation of permissible stresses and hence no rating of safety and lifetime is available.

To combine both methods, the standard requires a certain level of adaptation possibilities, to tune the major effects, which influence the stresses. Whereas ISO 10300 (edition 2014) has factors that allow an adaptation, the AGMA 2003:C10 standard has little possibilities. Also, the bending stress numbers of AGMA 2003 are much lower and differ remarkably from the contact analysis values.

The process to combine both calculation approaches increases the accuracy in the rating of bevel gears significantly. The first step is to determine the E, P, G, and Alpha displacements for a sample bevel-gear pair. By using the E, P, G, and Alpha displacements, the largest possible contact pattern is developed, strictly avoiding any edge contact. Based on the stress numbers by the contact analysis, the relevant parameters of the rating standard are derived.

As a next step, using the fast calculations based on the standards, the bevel-gear macro geometry is optimized by variation of the key parameters. All these solutions can be evaluated with various failures modes such as root bending, pitting, scuffing, and flank fracture.

1 INTRODUCTION

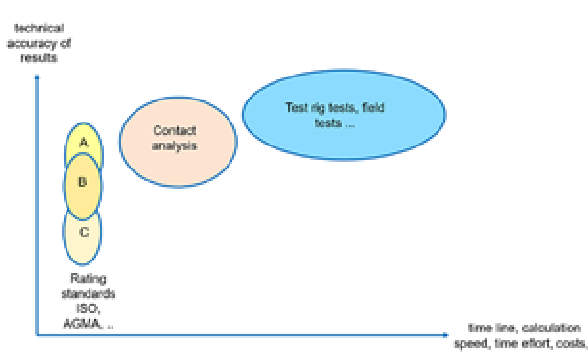
The design process for bevel gearsets is challenging. Many rating methods are available that have their specific possibilities, but they also may give different results in stresses or safety factors. The engineer is uncertain which method to trust and how to combine the various tools to achieve the most reliable result.

This article compares the various methods of ISO and AGMA standards, as well as the loaded contact analysis, using a practical sample.

2 STRENGTH RATING METHODS

In principle, for rating of bevel gears, several methods are available, such as rating standards and loaded tooth contact analysis (LTCA). These differ in available result types, level of detail, accuracy, and calculation time. That is why they are applied in various phases of the design. Figure 1 (left) shows a comparison between the methods in general.

The various methods provide different types of results. So, for example, the lifetime results are obtained only by rating standards or test rig trials. On the contrary, the LTCA provides the highest accuracy of stresses, as it is possible to consider the details such as manufactured modifications and exact misalignments. The misalignment values are obtained by a system design analysis, which delivers the results for



	Rating standard	Contact analysis	Test rig tests, field tests
Source	ISO, AGMA, ...	KISSsoft, GEMS, FEM, ..	Customer site
Life time information, safety value	✓	✗ (stresses, stress distribution)	✓
Macro geometry	✓ (varied, optimization)	✓ (defined, 1 gear pair)	✓ (defined)
Micro geometry (incl. tolerances & misalignments)	✗ (method A)	✓ (varied, optimization)	✓ (defined)

Figure 1: Available strength rating methods (left) and result types (right).

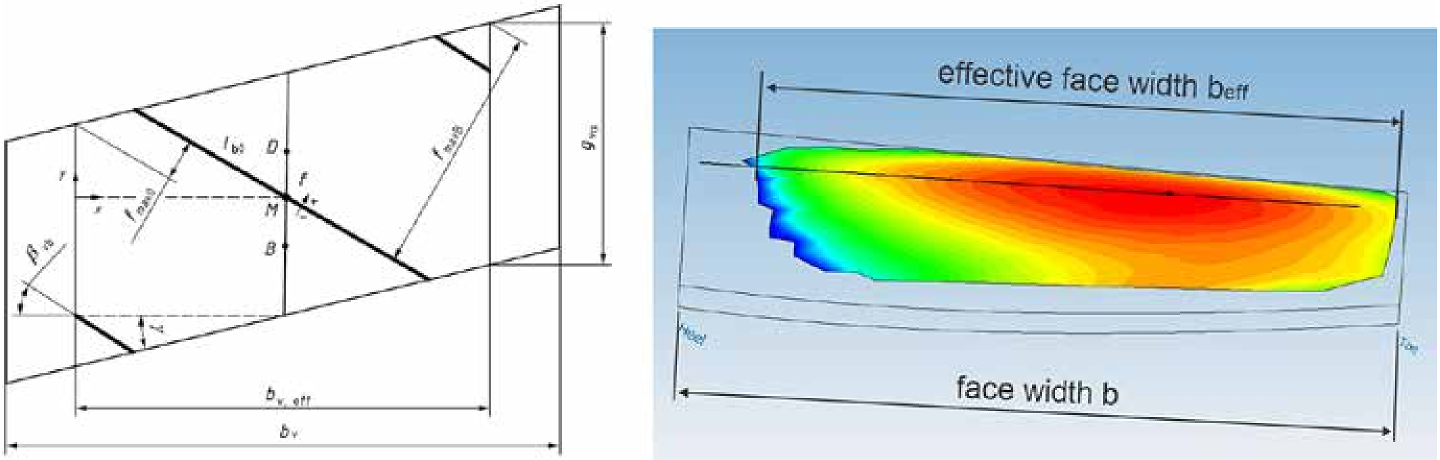


Figure 2: Calculation of contact pattern (left) and determination of beff using LTCA (right).

the three-dimensional position of the two meshing members due to the deflections of shafts, bearings, and the housing.

2.1 RATING STANDARDS

Strength rating of bevel gears according to standards such as AGMA, ISO, etc. are based on analytical (formula-based) calculations. Hereby, the bevel-gear geometry is transformed into a virtual cylindrical gear. This allows one to apply the same calculation approaches as used for cylindrical gears, which are modified by some specific bevel-gear factors, to consider the specific properties of bevel gears.

Verification of contact pattern	Mounting conditions of pinion and wheel		
Contact pattern is checked:	Neither member cantilever mounted	One member cantilever mounted	Both members cantilever mounted
for each gear set in its housing under full load	1,00	1,00	1,00
for each gear set under light test load	1,05	1,10	1,25
for a sample gear set and estimated for full load	1,20	1,32	1,50
NOTE Based on optimum tooth contact as evidenced by results of a contact pattern test on the gears in their mountings.			

Figure 3: Determination of mounting factor $K_{H\beta\text{-be}}$.

Rating standards deliver the occurring stresses (contact stresses, root stresses, etc.) as well as allowable stresses, which are used to calculate safety factors and lifetime results.

The rating by standards represents a simplified calculation approach but allows for a very quick evaluation of macro geometry. This is especially useful in the dimensioning phase, as it can be combined with "DOE" or "run-many-cases" methods.

2.1.1 ISO 10300

The rating standard of the International Organization for Standardization (ISO) is the ISO 10300 [1]. As of today, it includes the assessment of tooth bending, pitting, and scuffing for bevel and hypoid gears. Due to the five-year revision rule, the edition 2014 is currently under revision, and some minor modifications may be implemented. The rating calculations for the failure modes flank fracture and micro pitting are currently in a draft phase.

The ISO 10300 allows the engineer to modify various parameters to the current bevel-gear design and application. These must be understood by the design engineer, which is why these are explained in the following sections.

2.1.1.1 ADAPTATIONS FOR THE VIRTUAL CYLINDRICAL GEAR

For the calculation of the virtual cylindrical gear, the target is to calculate the contact pattern, including the contact lines, simulating the real contact of the bevel-gear set. The contact pattern is typically shorter than the nominal face width, due to the larger crowning value compared to cylindrical gears, to avoid edge contact under nominal load. The shortened contact pattern length is considered with the parameter "effective face width" b_{eff} (Figure 2, left). The contact pattern length is measured in the LTCA along the pitch cone (Figure 2, right).

The value for b_{eff} is a user input. As a default, the standard recommends using 0.85. However, the value of 0.85 is rather conservative. For today's optimized bevel-gear designs, the contact is typically larger, and it is state-of-the-art to determine the effective face width through LTCA tools or derive from test rigs.

For the contact ratio calculation, the ISO 10300 2014 edition includes the value for b_{eff}. In contrast, in the 2001 edition and also other standards, the full-face width is used for the calculation of

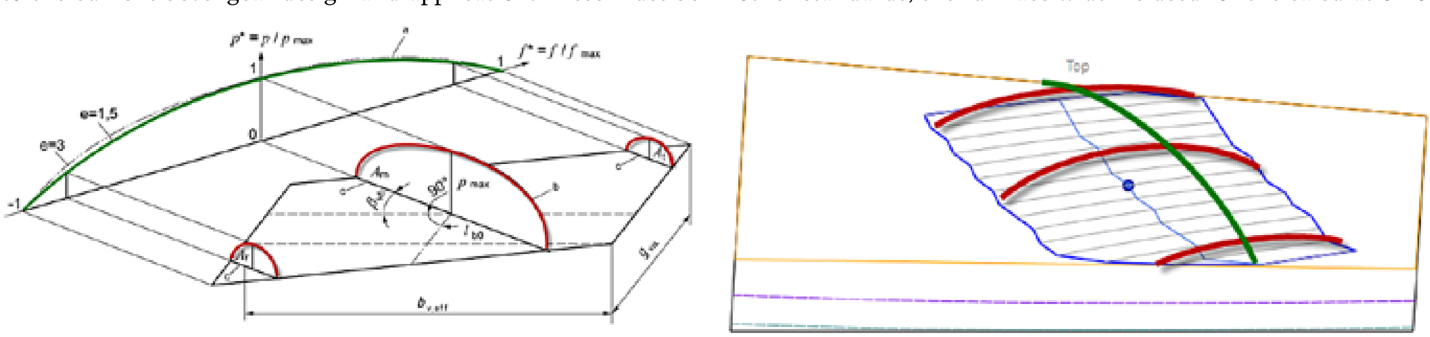


Figure 4: Definition of parabola along path of contact (left) and on tooth flank (right).

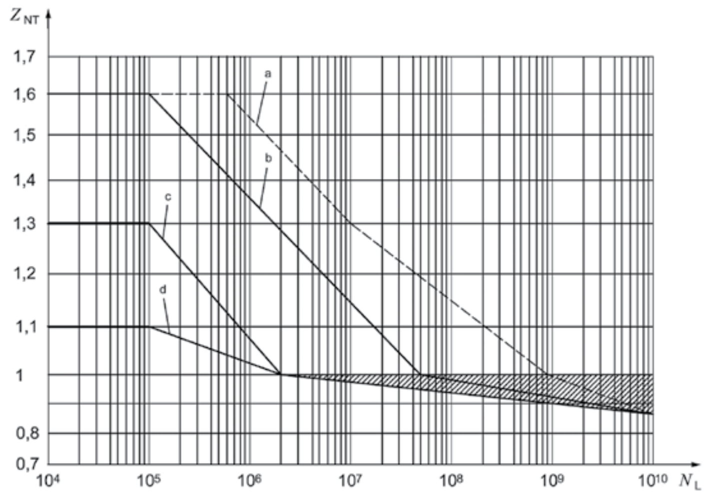


Figure 5: Life factor for pitting resistance Z_{NT}.

contact ratio. This leads to smaller contact ratio numbers in edition 2014 compared to others.

2.1.1.2 ADAPTATIONS FOR GENERAL FACTORS

The application factor K_A is defined as the ratio between the cyclic peak torque and the nominal rated torque. It considers the periodic load variation from both the input side (e.g. engine) as well as the output side (e.g. the drivetrain). Today, the factor is often replaced by a load spectrum (duty cycle) that allows a more precise load estimation. Still, the application factor is an easy but effective parameter to adopt to periodic overloads.

2.1.1.3 ADAPTATIONS FOR SURFACE DURABILITY

The contact stress calculates as follows in Equations 1 and 2:

$$\sigma_{\text{H0-B1}} = \sqrt{\frac{F_{\text{n}}}{l_{\text{bm}}\rho_{\text{rel}}}} Z_{\text{M-B}} Z_{\text{LS}} Z_{\text{E}} Z_{\text{K}} \qquad \textbf{Equation 1}$$

$$\sigma_{\text{H-B1}} = \sigma_{\text{H0-B1}} \sqrt{K_{\text{A}} K_{\text{v}} K_{\text{H}\beta} K_{\text{H}\alpha}} \qquad \textbf{Equation 2}$$

The bevel gear factor Z_K is an empirical factor that accounts for the differences between cylindrical and bevel gears in such a way as to agree with practical experience. It is a stress adjustment constant that permits the rating of bevel gears, using the same allowable contact stress numbers as for cylindrical gears. In the edition 2014, the bevel gear factor Z_K is 0.85.

The face load factor $K_{H\beta}$ reflects the non-uniform distribution of the load along the contact line. It is based on the mounting factor $K_{H\beta\text{-be}}$, multiplied by 1.5. The mounting factor depends on the mounting conditions of pinion and gear, as well as on the verification of contact pattern. In most transmissions, the pinion is cantilever mounted, whereas the gear is supported on both sides. The verification is done in many cases under light load only, which means in a roll tester. These two conditions result in a mounting factor of 1.1, which results in a $K_{H\beta}$ of 1.65 (Figure 3).

By using an LTCA, the load distribution along the contact line can be determined precisely. Typically, the load distribution for bevel gears is in a range of 1.5–1.7. So, using $K_{H\beta}$ with 1.65 is a good number for overload along face width.

The load sharing factor Z_{LS} considers the load distribution along the path of contact. The distribution of the peak loads per each contact line (red lines) along the path of contact (green line) is assumed to follow a parabola (Figure 4, left). This parabola can have two distributions, "automotive" and "industrial bevel gears." As the path of contact goes mainly in profile direction (Figure 4, right), the setting for Z_{LS} refers to the amount of profile crowning.

The allowable contact stress calculates as follows in Equation 3:

$$\sigma_{\text{HP-B1}} = \sigma_{\text{H,lim}}\ Z_{\text{NT}} Z_{\text{X}} Z_{\text{L}} Z_{\text{v}} Z_{\text{R}} Z_{\text{W}} Z_{\text{Hyp}} \qquad \textbf{Equation 3}$$

The life factor Z_{NT} accounts for the higher contact stress, including static stress, which may be acceptable for a limited life (number of load cycles), as compared with the allowable stress where $Z_{NT} = 1.0$. For extended life, Z_{NT} may be less than 1.0. Z_{NT} has been determined for standard test gear conditions.

The engineer must decide about the extended life. The standard provides two options: The first option is to have $Z_{NT} = 1$ from the "knee" to 10^{10} load cycles. This is applicable for optimum lubrication, material, manufacturing, and experience. The second option is to reduce the allowable stress by $Z_{NT} = 0.85$ from the "knee" to 10^{10} load cycles (Figure 5).

2.1.1.4 ADAPTATIONS FOR ROOT BENDING STRENGTH

The root bending stress calculates as follows in Equations 4 and 5:

$$\sigma_{\text{F0-B1}} = \frac{F_{\text{vmt}}}{b_{\text{v}}\, m_{\text{mn}}} Y_{\text{Fa}} Y_{\text{Sa}} Y_{\varepsilon} Y_{\text{BS}}\, Y_{\text{LS}} \qquad \textbf{Equation 4}$$

$$\sigma_{\text{F-B1}} = \sigma_{\text{F0-B1}} K_{\text{A}}\, K_{\text{v}}\, K_{\text{F}\beta}\, K_{\text{F}\alpha} < \sigma_{\text{FP-B1}} \qquad \textbf{Equation 5}$$

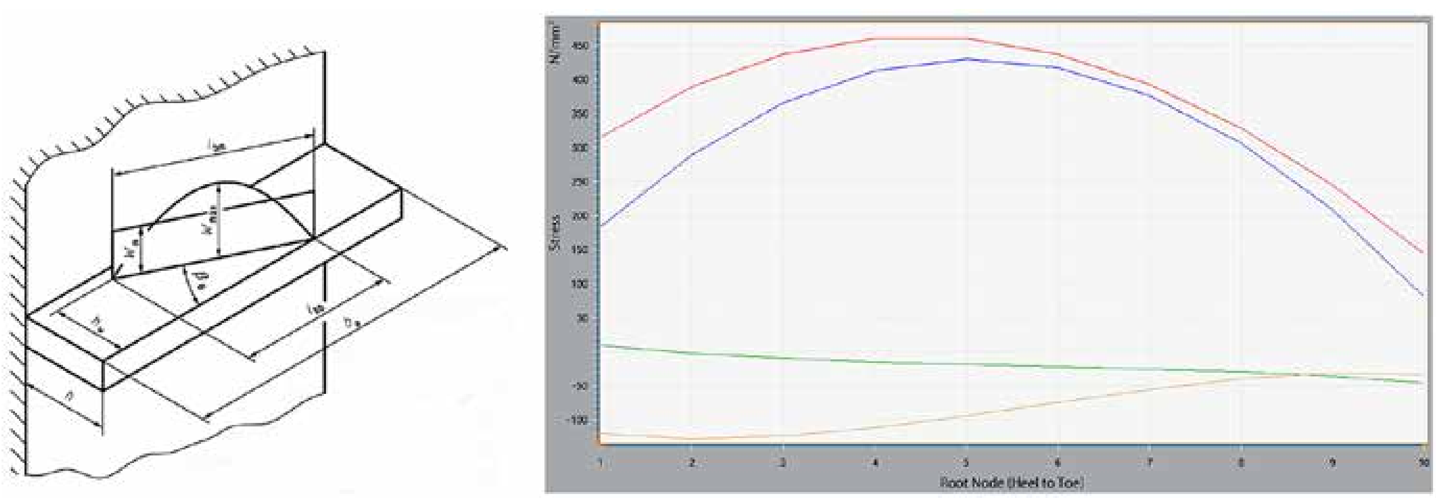


Figure 6: Calculation of bevel spiral angle factor Y_{BS} (left) and tooth root stress distribution by LTCA (right).

The load sharing factor Y_{LS} accounts for load sharing between two or more pairs of teeth. It is calculated from Z_{LS}.

The bevel spiral angle factor Y_{BS} accounts for smaller values for contact lines l_{bm} compared to the total face width and the inclined lines of contact. The bevel spiral angle factor replaces the bevel gear factor Y_K from edition 2001, which stands for differences of length of contact lines between cylindrical and bevel gears.

The stress distribution in the tooth root depends on the inclination of the contact lines. With an increased spiral angle, the contact lines are limited by tip and root of the tooth (Figure 6, left). This leads to a higher stress maximum in the tooth root in the middle of the face width. With LTCA, the tooth root distribution is calculated based on the inclination of the contact line and the exact load distribution along the contact line (Figure 6, right).

The permissible bending stress calculates as follows in Equation 6:

$$\sigma_{FP} = \frac{\sigma_{F\,lim} \cdot Y_{ST} \cdot Y_{NT}}{S_{F\,min}} \cdot Y_{\delta\,rel\,T} \cdot Y_{R\,rel\,T} \cdot Y_X \qquad \text{Equation 6}$$

For the life factor Y_{NT}, the same applies as for the life factor Z_{NT}.

2.1.2 AGMA 2003

The AGMA 2003 is the rating standard provided by the American Gear Manufacturers Association [2]. The edition C10 from year 2010 is used for these calculations. It includes the root bending stress and pitting calculation. The equations are shown in two formats. In the following, the equations are shown in SI units and ISO symbols.

The contact stresses and permissible stresses are calculated as follows in Equations 7 and 8:

$$\sigma_H = Z_E \sqrt{\frac{2000T_1}{bd_{e1}^2 Z_I} K_A K_v K_{H\beta} Z_x Z_{xc}} \qquad \text{Equation 7}$$

$$\sigma_{HP} = \frac{\sigma_{H\lim} Z_{NT} Z_W}{S_H K_\theta Z_Z} \qquad \text{Equation 8}$$

2.1.2.1 ADAPTATIONS FOR CONTACT STRESS

The overload factor K_A (K_O) recommendation is similar to ISO 10300-1, whereas the "character of prime mover" and "character of load on driven machine" are to be considered.

The dynamic factor K_V is based on the parameters pitch line velocity and transmission accuracy, Q_v. When manufacturing techniques ensure equivalent transmission accuracy, Q_v can be the same as the quality number for the lowest quality member in mesh. This means the transmission accuracy is related directly to the gear quality.

The load distribution factor $K_{H\beta}$ (K_m) refers to the load distribution modifier K_{mb}, which is defined by the user (Table 1). The final load distribution is much lower than that, according to ISO 10300, as it is added by the term 5.6*10-6*b^2 only. This results in a load distribution factor $K_{H\beta}$, which is only slightly larger than the load distribution modifier alone.

Mounting factor according to AGMA 2003			
	Mounting conditions of pinion and gear		
	both members straddle mounted	one member straddle mounted	neither member straddle mounted
Mounting factor	1.00	1.10	1.25

Table 1: Load distribution modifier (mounting factor) according to AGMA 2003.

Requirements of application	Reliability factors for steel[1)] C_R (Z_Z)	Reliability factors for steel[1)] K_R (Y_Z)[2)]
Fewer than one failure in 10 000	1.22	1.50
Fewer than one failure in 1000	1.12	1.25
Fewer than one failure in 100	1.00	1.00
Fewer than one failure in 10	0.92	0.85[3)]
Fewer than one failure in 2	0.84	0.70[4)]

NOTES:
[1)] At the present time there are insufficient data concerning the reliability of bevel gears made from other materials.
[2)] Tooth breakage is sometimes considered a greater hazard than pitting. In such cases a greater value of K_R (Y_Z) is selected for bending.
[3)] At this value plastic flow might occur rather than pitting.
[4)] From test data extrapolation.

Table 2: Reliability factors for steel.

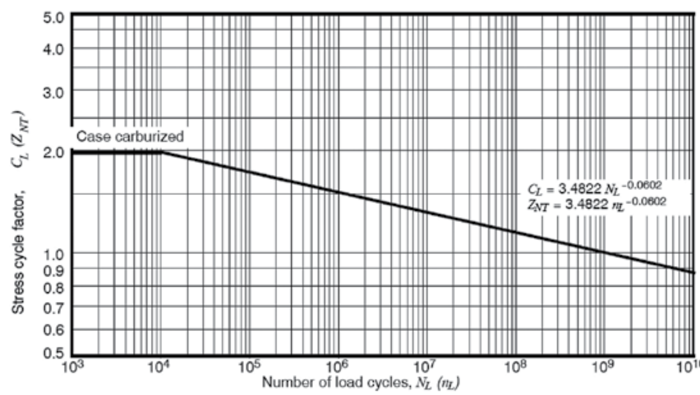


Figure 7: Stress cycle factor for pitting resistance Z_{NT}.

2.1.2.2 ADAPTATIONS FOR PERMISSIBLE CONTACT STRESS NUMBER

The reliability factor Z_Z accounts for the effect of the normal statistical distribution of failures found in materials testing (Table 2). The allowable stress numbers given are based upon a statistical probability of one failure in 100 as a unity life factor. (Equation 9)

$$K_{H\beta} = K_{mb} + 5.6 \times 10^{-6}\, b^2 \qquad \text{Equation 9}$$

The ISO 6336-5 uses a 1% probability of damage. So, the option "Fewer than one failure in 100" leads to an identical material reliability per the ISO.

The stress cycle factor for pitting resistance Z_{NT} (C_L) is defined for carburized case-hardened steel only. The S-N curve shows a constant decrease of the allowable contact stress number after the static area (Figure 7). There is no distinction between limited life and extended life existing, which represents the characteristics of the Corten/Dolan fatigue life prediction theory.

The allowable contact stress number σ_{Hlim} (s_{ac}) is defined in the standard AGMA 2003 directly. For steel gears, 3 grades are provided, whereas for carburized and case-hardened steels, the σ_{Hlim} values 1,380 N/mm^2 (grade 1), 1,550 N/mm^2 (grade 2) and 1,720 N/mm^2 (grade 3) are defined.

To find a correlation between the steel qualities of AGMA 2003 and ISO 6336-5, Figure 9 in ISO 6336-5 can be taken as reference. The MQ

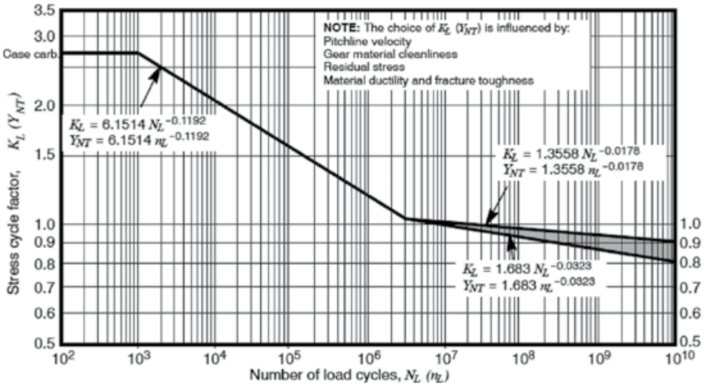


Figure 8: S-N curve for bending strength.

quality has σ_{Hlim} value of 1,500 N/mm^2. For pitting resistance, the steel quality MQ matches closest with the steel grade 2 of AGMA 2003.

2.1.2.3 BENDING STRESS CALCULATION

The bending stress σ_F and allowable bending stress σ_{FP} is calculated with Equations 10 and 11:

$$\sigma_F = \frac{2000\,T_1}{b\,d_{e1}} \frac{K_A\,K_v}{m_{et}} \frac{Y_x\,K_{H\beta}}{Y_\beta\,Y_J} \quad \text{Equation 10}$$

$$\sigma_{FP} = \frac{\sigma_{F\,\lim} Y_{NT}}{S_F K_\theta Y_z} \quad \text{Equation 11}$$

2.1.2.4 ADAPTATIONS FOR BENDING STRESS CALCULATION

For bending stress calculation, the same adoptions as for contact stress calculation apply.

2.1.2.5 ADAPTATIONS FOR ALLOWABLE BENDING STRESS CALCULATION

The stress cycle factor for bending strength Y_{NT} (K_L) has different characteristics than the factor of pitting resistance Z_{NT}. The extended life has a different slope than the limited life, which is similar to the ISO 6336-5. Also, the AGMA standard provides two options to be selected from. The first option is to reduce Z_{NT} = 0.9 until 10^{10} load cycles. The second option is to reduce the allowable bending stress from 1.0 at the knee down to Z_{NT} = 0.8 at 10^{10} load cycles (Figure 8).

The allowable bending stress number σ_{Flim} (s_{at}) is also defined in the standard AGMA 2003 directly. For steel gears, three grades are provided, whereas for carburized and case-hardened steels, the σ_{Flim} values are provided with 205 N/mm^2 (grade 1), 240 N/mm^2 (grade 2), and 275 N/mm^2 (grade 3) (Figure 9, middle).

Compared to the steel qualities ML, MQ, and ME of ISO 6336-5 (Figure 9, left), the numbers in AGMA 2003 are much lower, and the engineer has no guidance as to which grade of the AGMA corresponds with the material quality of the ISO. To find the correlation, the stress numbers of the cylindrical gear calculation (AGMA 2001-D04) can be taken as reference.

In the AGMA 2001-D04 for cylindrical gears, the σ_{Flim} values are provided with 380 N/mm^2 (grade 1), 450 N/mm^2 (grade 2) and 517 N/mm^2 (grade 3) (Figure 9, right). Herewith, the closest comparable steel from AGMA is grade 2 to ISO quality MQ(b) with σ_{Flim} = 460 N/mm^2.

2.2 RATING BY GLEASON Q-FACTOR

The Q-factor allows a rating of the bending stress. It was developed by Wells Coleman [3] and is also used in a similar form as the AGMA calculation. It includes the parameters P_d, K_s, F, D, and J. Today, Gleason uses a version that was improved by Coleman in 1981, which also allows the calculation for hypoid gears. (Equation 12)

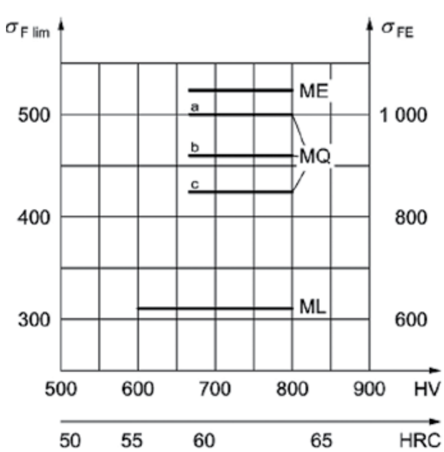
$$Q = \frac{2P_d \quad K_s}{F\,D\,J} \quad \text{Equation 12}$$

At that time of publication in 1981, no rating method for hypoid gears was available. The Q-factor was the only available parameter, and hence it is based on a certain experience basis (Design History Database). This is also the reason the Q-factor is still applied today by most automotive companies worldwide.

2.3 LOADED CONTACT ANALYSIS

With dedicated tools employing Finite Element or Boundary Element methods, the stresses for bevel and hypoid gears can be calculated using a loaded contact analysis (LTCA). This analysis allows the engineer to consider the individual flank modifications such as crowning, twist, etc., which are needed to achieve an optimized contact under various load conditions. To get closer to real world conditions in gearboxes, the displacements between the pinion and ring gear are also considered.

Both the no-load TCA and the loaded TCA take the exact flank topology into account created during the manufacturing process, based on the machine settings and tool geometry. This allows the engineer to analyze the strength as well as the rolling performance



ISO 6336-5

Bending stress number (allowable), s_{at} ($\sigma_{F\,lim}$) lb/in^2 (N/mm^2)		
Grade 1[1)]	Grade 2[1)]	Grade 3[1)]
See figure 10	See figure 10	
12 500 (85) 22 500 (154)	13 500 (95) --	-- --
30 000 (205)	35 000 (240)	40 000 (275)

AGMA 2003-C10

Allowable bending stress number[2)], s_{at} lb/in^2		
Grade 1	Grade 2	Grade 3
see figure 9	see figure 9	--
45 000	55 000	--
22 000	22 000	--
55 000	65 000 or 70 000[6)]	75 000

AGMA 2001-D04

Figure 9: Allowable stress numbers as per ISO and AGMA (cylindrical and bevel gears).

and NVH behavior of the gear set.

A principal lack of general finite element tools is the missing permissible stress calculations, and, therefore, the rating of safety and lifetime is not available. To improve that situation, Gleason developed a method where the root and contact stresses are compared with permissible stresses based on material properties. This allows an estimation of lifetime comparable to ISO or AGMA.

2.3.1 STRESS CALCULATION BY CUSTOM FE METHODS

For all LTCA analyses, the GEMS® FEA App [4] was employed, which is a gear-strength analysis program based on the finite element method. It combines the User Interface part "FEAV is Finite Element Visualization," which includes the post processor and an FEA preprocessor and solver DLL called "FEA Server — Finite Element Analysis Server." The mesh generation is based on a 3D 8-node structure solid element type. The resolution can be changed upon the required accuracy level as well as calculation speed and time. The mesh parameter can be set up for thickness, height, and root direction individually.

The user defines a finite element mesh based on the real blank geometry. The evaluation includes a large variety of stress analysis options, as well as a motion error analysis. The operating load can be defined by duty cycles. This represents the state-of-the-art in rating of transmissions, also including the bevel- and hypoid-gear stages. In addition to the damage accumulation, the varying loads also lead to different misalignments. This allows a complete rating regarding stresses, deflections and life expectancy of bevel and hypoid gear designs.

2.3.2 MISALIGNMENT VALUES USING E, P, G, AND ALPHA PARAMETERS

Under load, the pinion and ring gear are misaligned due to the reaction forces and stiffness of the shafts, bearings, and the housing structure. The misalignments are defined by the four parameters E, P, G, and Alpha, which are accumulated from the pinion and ring-gear deflections. E means the misalignment in offset direction; P is the misalignment in pinion axis direction; G the misalignment in ring gear axis direction, and Alpha is the angular misalignment in the plane of shaft axis angle.

The parameters are defined in the crossing point of the bevel gearset (Figure 10, left). The calculation of the misalignments can be easily expressed by the vector approach (Figure 10, right).

2.4 COMBINED APPROACH

Although the calculation of stresses is available by analytical and numerical approaches today, it is not yet state-of-the-art to combine the two methods. On one hand, the rating standards require a certain level of adaptation possibilities to tune the major parameters to achieve accurate results similar to the LTCA.

The process to combine both calculation approaches increases the accuracy in the design and simulation of bevel gears significantly. When using the LTCA, the E, P, G, and Alpha displacements are used to develop the largest possible contact pattern by strictly avoiding edge contact. This consequently results in an optimized contact. Based on the stress numbers obtained by the LTCA, the rating standard can be tuned to deliver more precise stress results. Together with the allowable stresses, the resulting lifetimes and safety factors are calculated precisely.

3 SAMPLE DESIGN

3.1 TRANSMISSION

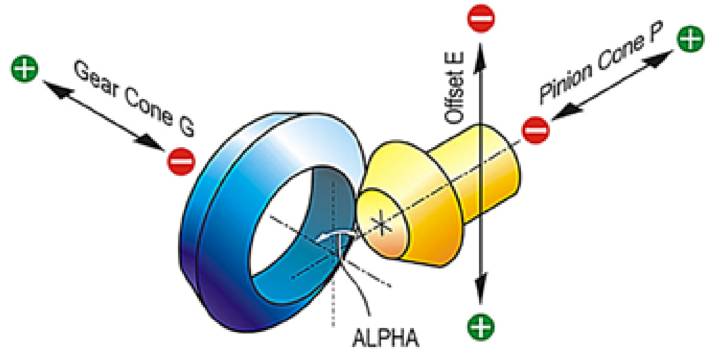
As a sample design, a bevel gearset of a tractor rear axle is used. The drivetrain is modeled in KISSsys [5] and contains the complete drivetrain, including the manual transmission, consisting of a high-speed (H) and low-speed gear train (L), having four speeds each. To achieve further reduction of speed, a planetary axle drive set was applied (Figure 11).

The bevel gearset of the rear axle was designed using an outer ring gear diameter of 310 mm with 35 gear teeth and 11 teeth on the mating pinion, resulting in a ratio of 3.2. The nominal operating conditions are 1,500 Nm and 165 RPM at the bevel pinion.

For the same, a load spectrum (duty cycle) was applied in the drivetrain design software with a total of 36 load cases, for all speeds. For the duty cycle, a maximum pinion torque of 2,400 Nm was used.

3.2 MISALIGNMENT VALUES

In the drivetrain design software, the misalignment values E, P, G ,and Alpha are calculated for the drive side with E = –0.3 mm, P = 0.14 mm, G = –0.23 mm. For the coast side, the values are determined with E = 0.41 mm, P = –0.17 mm, G = –0.50 mm (Figure 12). These values were used in the loaded tooth contact analysis for the development of the contact pattern.



$$\mathbf{S} = a\cos\left(\frac{\bar{v}_1 \bullet \bar{v}_2}{\|\bar{v}_1\|\cdot\|\bar{v}_2\|}\right)\cdot\frac{180}{\pi} - \Sigma \qquad \mathbf{E} = \frac{\left|(\bar{x}_1 - \bar{x}_2)\bullet(\bar{v}_1 \times \bar{v}_2)\right|}{\|\bar{v}_1 \times \bar{v}_2\|} - a$$

$$\frac{\left\|(\bar{x}_{p1} - \bar{x}_2)\times\bar{v}_2\right\|}{\|\bar{v}_2\|} = E + a \quad \text{, where } \bar{x}_{p1} = \bar{x}_1 + (P_{nom} + \mathbf{P})\cdot\bar{v}_1$$

$$\frac{\left\|(\bar{x}_{p2} - \bar{x}_1)\times\bar{v}_1\right\|}{\|\bar{v}_1\|} = E + a \quad \text{, where } \bar{x}_{p2} = \bar{x}_2 + (G_{nom} + \mathbf{G})\cdot\bar{v}_2$$

Figure 10: Definition of E, P, G, and Alpha (S) value (left) and calculation in vector approach (right).

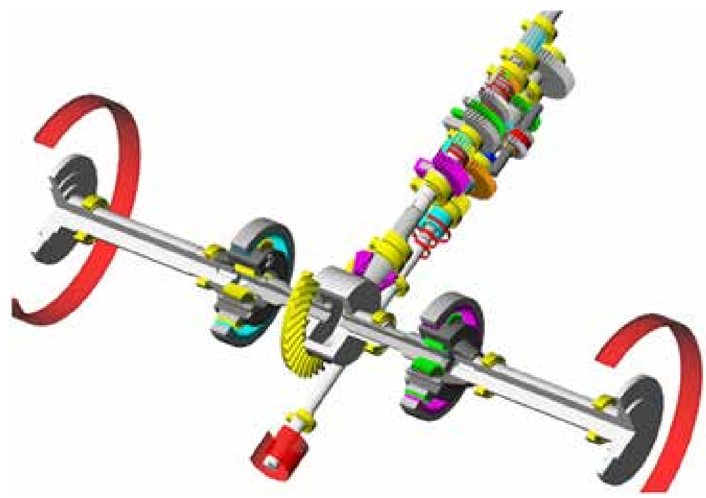
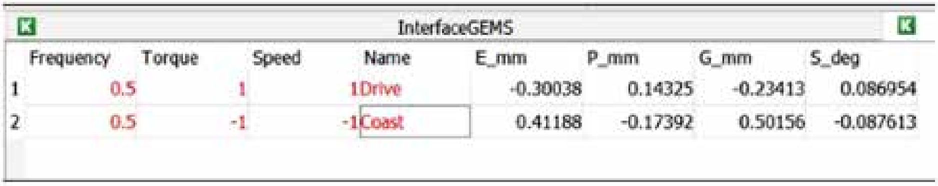


Figure 11: Transmission and rear axle of a tractor.



	Frequency	Torque	Speed	Name	E_mm	P_mm	G_mm	S_deg
1	0.5	1	1	Drive	-0.30038	0.14325	-0.23413	0.086954
2	0.5	-1	-1	Coast	0.41188	-0.17392	0.50156	-0.087613

Figure 12: E, P, G, and Alpha values for drive and coast side.

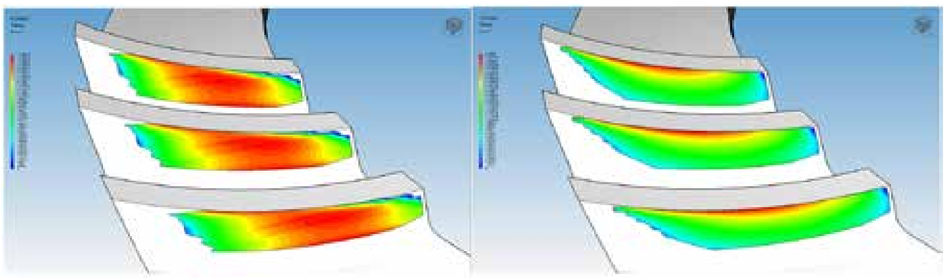


Figure 14: Contact pattern under load without misalignment (left) und with misalignment (right).

3.3 DEVELOPMENT OF CONTACT PATTERN

In a first step, the macro geometry and the E, P, G, and Alpha values were transferred using the xml-based Design Data Exchange interface between KISSsoft and GEMS. Within GEMS, from a basic design perspective, the first iteration was optimized for equal stress conditions, changing tooth thickness, and cutter edge radii. In Figure 13, the first no load TCA is shown before the optimization for E, P, G, and Alpha deflections. The mean contact pattern was pre-positioned to the toe, and the length crowning was optimized for the nominal torque to reach 80-percent contact length under load. Together with a moderate profile crowning and flank surface bias, a motion error of 40 micro radians under zero load condition was achieved.

The contact pattern under load with zero deflection showed an ideal position but did not represent the actual position under deflections of the transmission under load (Figure 14, left). With the misalignments considered, the contact pattern moved toward the tip, which required further contact pattern development (Figure 14, right).

Thereafter, the contact pattern was developed considering the E, P, G, and Alpha values under nominal load. For the lengthwise position, the contact pattern was developed under nominal load for largest spread in face with direction to achieve the highest possible load distribution and the lowest contact stresses. The observation of the contact pattern in the profile direction showed that, under load, the contact moves toward the tip of the ring gear teeth. In order to achieve a nice and centered tooth contact under load, additional reliefs in form of Flankrem on the ring gear and Toprem® on the pinion were necessary (Figure 15).

The contact pattern position was also calculated under light load (bench contact) using the corresponding lower misalignment values (Figure 16, right). The optimized design shows a good compromise between light load and nominal load (Figure 16, left).

3.4 EVALUATION OF STRESSES

3.4.1 CALCULATION OF STRESSES BY LTCA

Using the LTCA, the stress values depend on the real contact pattern position, which is more realistic compared to the simplified standards. Additionally, when varying the pinion mounting distance (caused by, for example, tolerances of the shim washer, housing, etc.), the

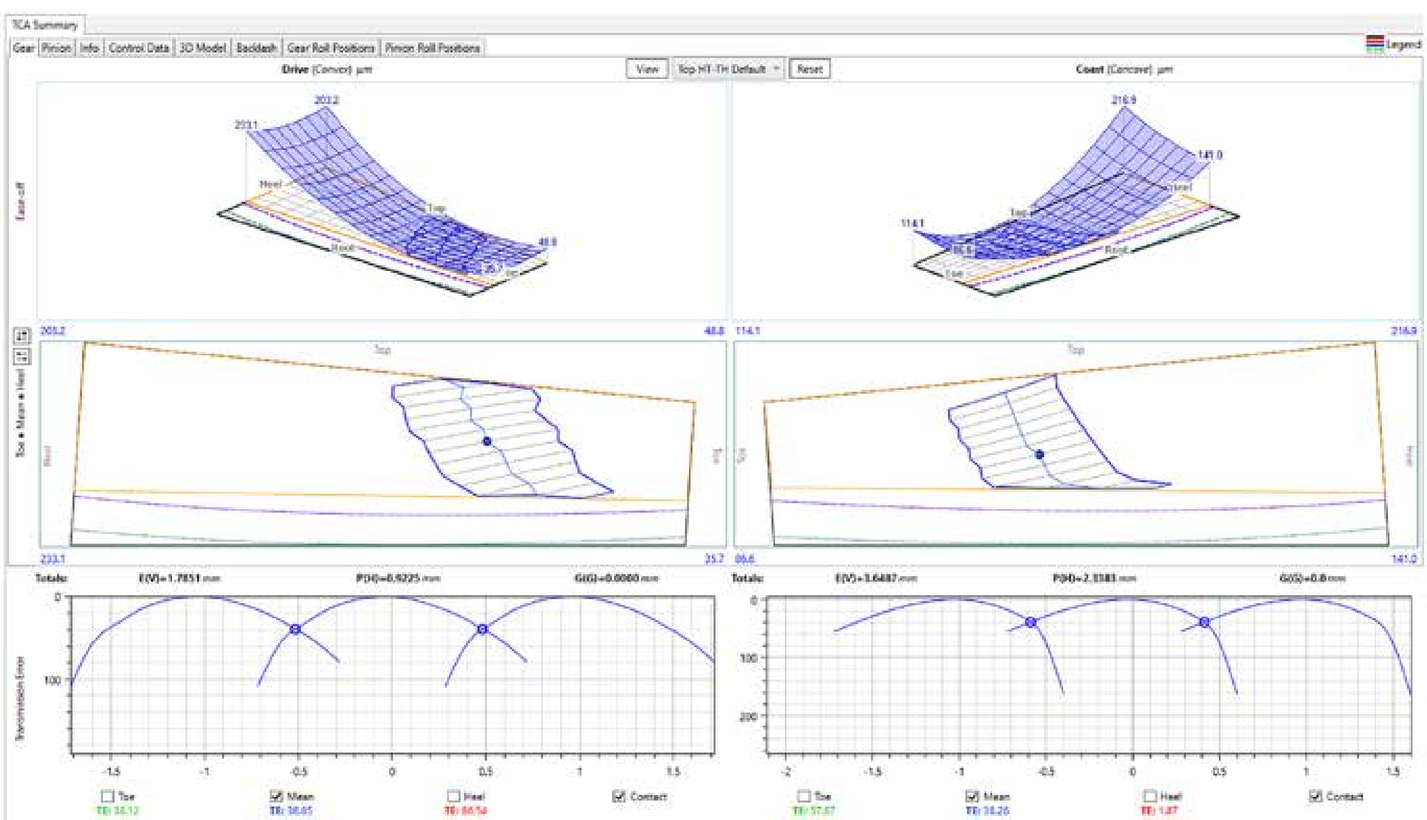


Figure 13: Initial contact pattern development without E, P, G, and Alpha.

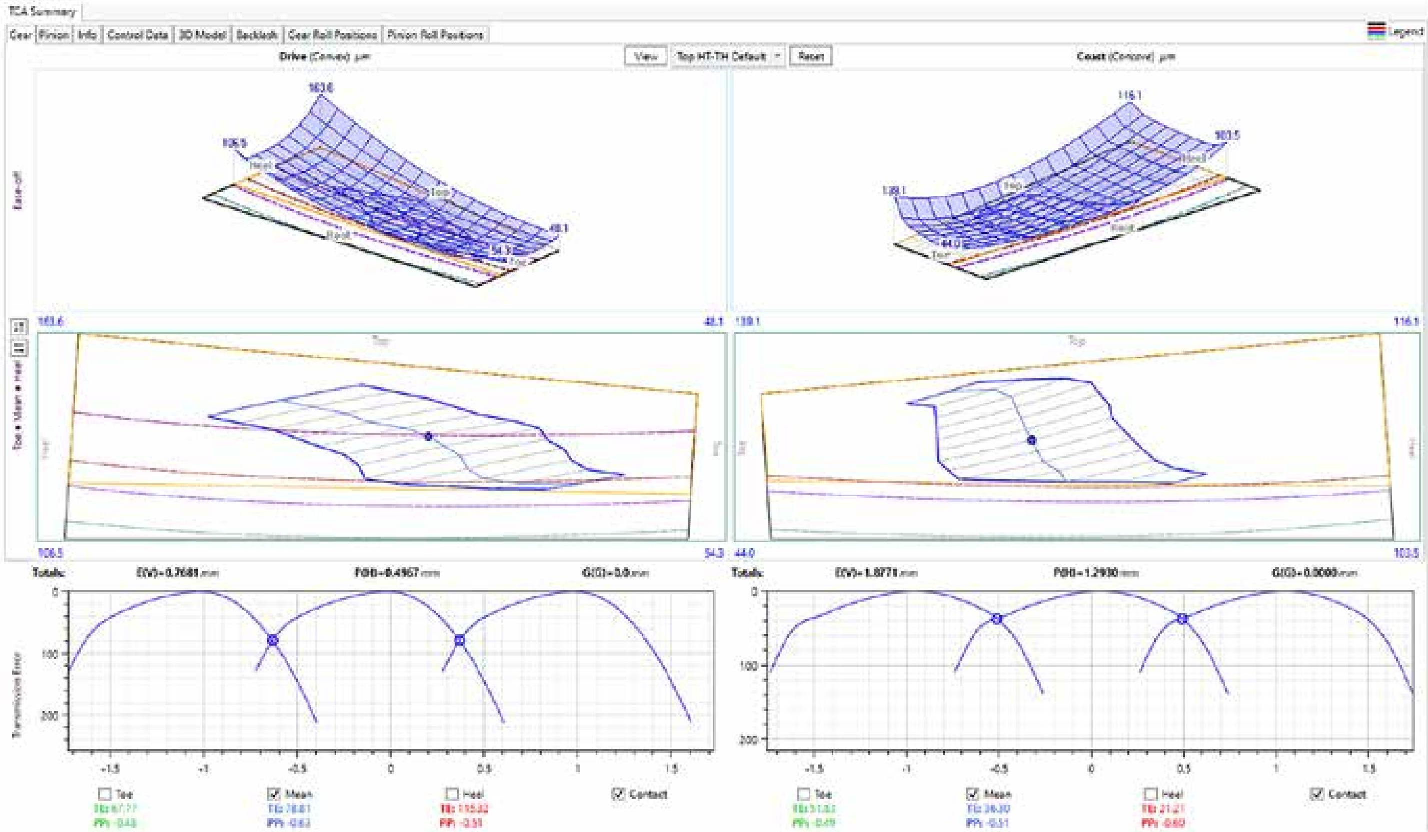


Figure 15: Contact pattern development including Flankrem and Toprem®.

simulation shows different contact patterns and stresses. Table 3 shows various contact patterns under load and the corresponding stress numbers for contact and root stresses. The backlash was kept constant by adjusting the ring gear mounting distance via the G-value.

It shows that, with decreasing pinion mounting distance (H), the contact stress drops overall by 17 percent. The root stresses vary within 12 percent. However, the assessment of the best pinion mounting distance is not a matter of the contact stress number only. The visual assessment of the contact pattern is even more important. The contact pattern at H = 0 mm shows no edge load, whereas the contact pattern with negative mounting distance shows a contact stress concentration at the heel edge. This is not acceptable as an assembly position. Also looking at the peak-to-peak transmission error under load, it can be confirmed that the contact pattern is best at H = 0 mm.

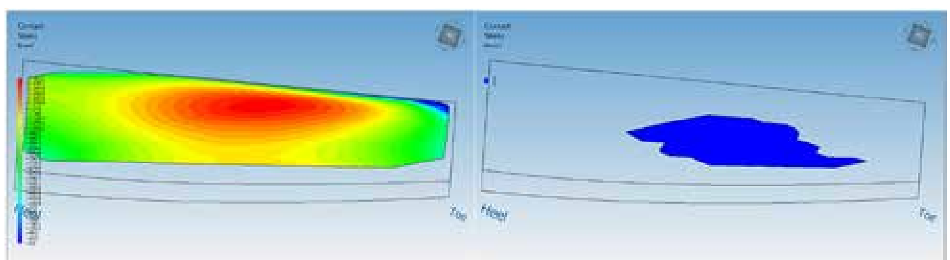


Figure 16: Contact stress under nominal load (left) and bench contact (light load, right).

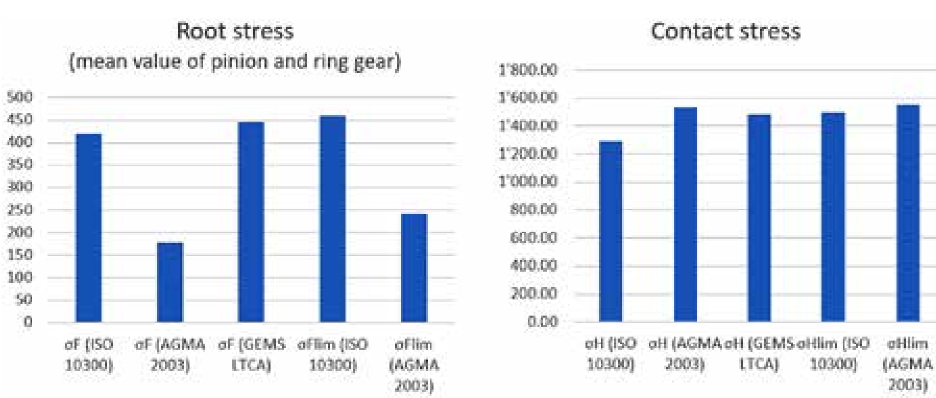


Figure 17: Comparisons of ISO, AGMA, and LTCA results for root (left) and contact stresses (right).

3.4.2 CALCULATION OF STRESSES BY ISO 10300

The effective face width b_{eff} is measured from the contact pattern in GEMS with 0.92. The profile crowning is considered as rather low, which is common in automotive and truck applications. As face load distribution, the mounting factor $K_{H\beta\text{-}be}$ is = 1.1, which results in $K_{H\beta}$ = 1.65. As root fillet radius, the blade edge radius of the tool was used, as it was developed in GEMS.

The calculated contact stress was found with σ_H = 1,244 N/mm^2, the root stress was calculated with σ_F = 419 N/mm^2 (mean value between pinion and ring gear).

The steel was selected with 18CrNiMo7-6, the steel quality was MQ with a core hardness ≥25 HRC and σ_{Flim} = 460 N/mm^2 and σ_{Hlim} = 1,500 N/mm^2.

3.4.3 CALCULATION OF STRESSES BY AGMA 2003

For surface load distribution, the load modifier K_{mb} ($K_{H\beta\text{-}be}$) is = 1.1, which results in a $K_{H\beta}$ of 1.15. The root radius is used from the blade

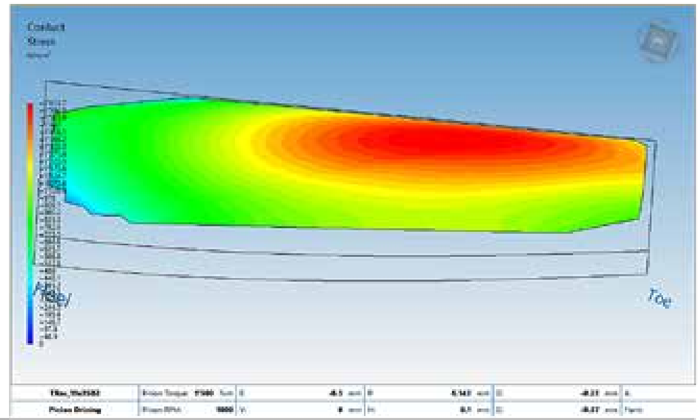
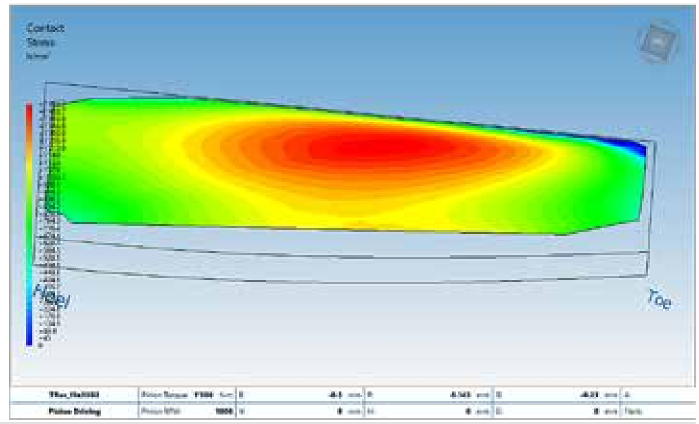
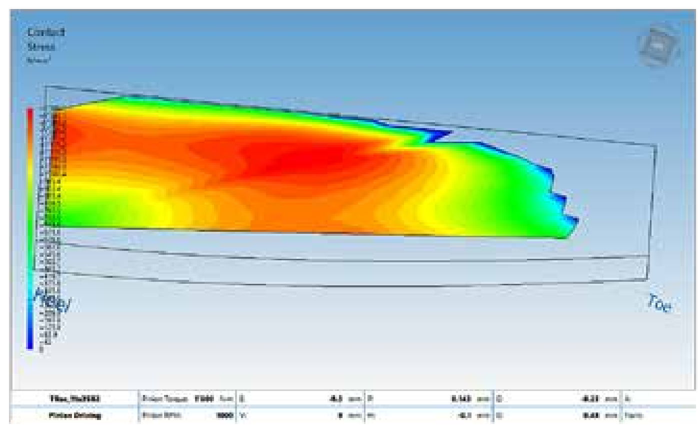
Contact pattern at H = +0.1 mm Contact stress = 1614 N/mm^2 Bending stress pinion = 413 N/mm^2 Bending stress gear = 479 N/mm^2 Peak-to-peak transmission error = 306.6 µrad	
Contact pattern at H = 0 mm Contact stress = 1484 N/mm^2 Bending stress pinion = 461 N/mm^2 Bending stress gear = 430 N/mm^2 Peak-to-peak transmission error = 167.2 µrad	
Contact pattern at H = −0.1 mm Contact stress = 1385 N/mm^2 Bending stress pinion = 420 N/mm^2 Bending stress gear = 482 N/mm^2 Peak-to-peak transmission error = 360.2 µrad	

Table 3: Stress numbers depending on mounting position of the pinion.

edge radius in GEMS. The calculated contact stress was found with σ_H = 1530 N/mm^2, and the root stress was calculated with σ_F = 178 N/mm^2 (mean value between pinion and ring gear).

The steel was selected with grade 2, with σ_{Flim} = 241.32 N/mm^2 (35,000 lb/in^2) and σ_{Hlim} = 1,551.32 N/mm^2 (225,000 lb/in^2).

3.5 COMPARISON OF STRESSES

The stresses are compared between the rating standards ISO 10300, AGMA 2003, and the loaded contact analysis.

The tooth root stresses between ISO and LTCA match very well. This is a trustworthy result, even if the fact is considered that the ISO standard counts the root stresses at the 30° tangent, whereas the LTCA may not have the maximum stresses at the same point. The root stresses σ_F and σ_{Flim} of the AGMA standard are much lower. This is unexplained and does not correspond with ISO and LTCA simulation (Figure 17, left).

The contact stresses as well as the allowable stresses match quite well between AGMA and LTCA. However, the ISO result shows 13 percent lower contact stress than the LTCA and the AGMA method (Figure 17, right).

Strength Factor Q		
	Pinion	Gear
Gleason (GEMS)	2.42775	0.76967
AGMA (KISSsoft)	2.42446	0.76832
Delta [%]	-0.14%	-0.18%

Table 4: Comparison of Q-factor.

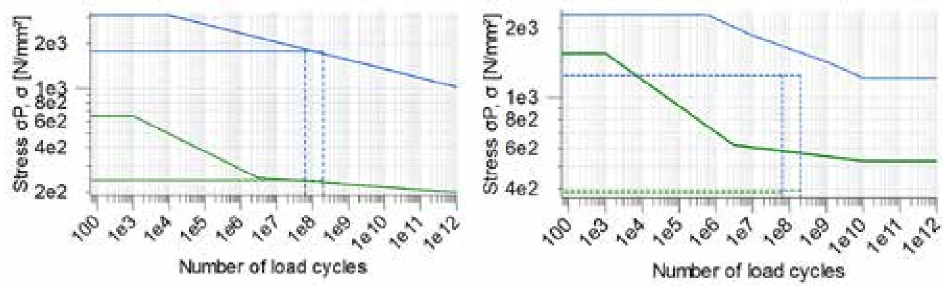


Figure 18: S-N curves and load bin (case, single-stage) for AGMA (left) and ISO (right).

3.6 COMPARISON OF Q-FACTOR VALUES

The Q-factors were calculated based on the AGMA rating standard using KISSsoft and compared to the approach implemented in the Gleason GEMS software. The results match very well and are almost identical (Table 4). This means the rating of a bevel gear set by the Q-factor with the AGMA 2003 within KISSsoft is analog to the GEMS calculation. This allows the engineer to use the Q-factor already in a dimensioning step, e.g., when running many cases.

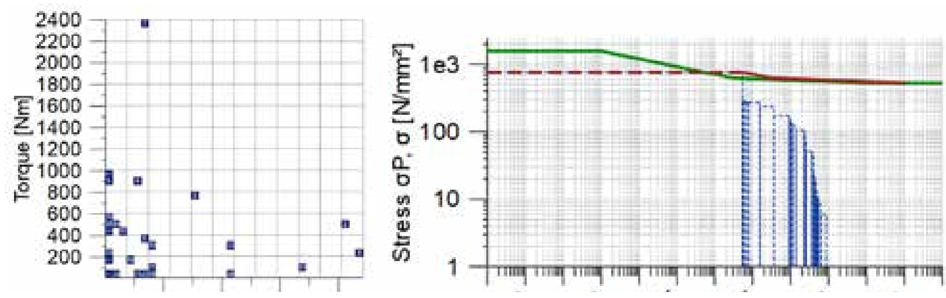


Figure 19: Torque over frequency of bins (left) and S-N curve of load spectra.

3.7 EVALUATION OF SAFETY FACTORS

For the calculation of safety factors, the bevel-gear design with tooth thickness modifications according to the “equal stress” method was taken. Herewith, the tooth root safety factors are balanced between pinion and ring gear, considering slightly larger values on the ring gear than on the pinion.

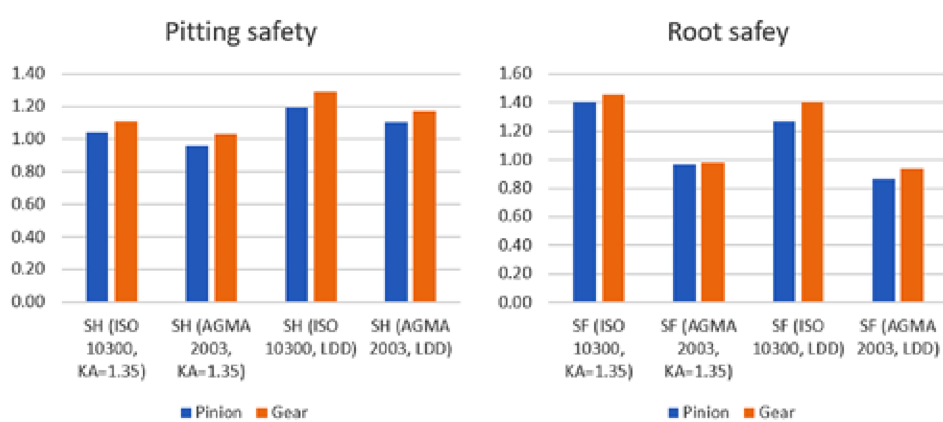


Figure 20: Comparison of pitting (left) and root safety numbers (right).

3.7.1 CALCULATION SETTINGS

For the calculation of safety factors and lifetime using the ISO and AGMA standards, two load scenarios were investigated, one with nominal load and one with duty cycle.

For the calculation with nominal load, the application factor was chosen with 1.35. This is a realistic overload factor for a tractor transmission. Comparing the S-N curves, it becomes obvious that the number of load cycles are in the extended (long) life section (Figure 18). The settings for the S-N curve for allowable root stress between AGMA (left) and ISO (right) are defined as close as possible, to have highest comparability in this study (green color). Also, the much lower allowable root stress of AGMA (left) becomes obvious.

For the analysis with duty cycle, a duty cycle with 36 bins (load cases) was created in the drivetrain design software, which considers all speeds and torques for high and low range (from L1 to H4, totally 8 speeds) typical load scenarios. The 36 bins are applied to the bevel gearset, which is a final drive stage. The nominal load of T = 2,400 Nm was applied, which represents the highest possible torque at the first speed. A duty cycle analysis allows a much more detailed look at the damage accumulation.

The frequency distribution (torque over frequency) shows the 36 bins and the corresponding torques (Figure 19, left). The S-N curve also shows the 36 bins (Figure 19, right). For the first speed (L1), only one bin was applied. It shows clearly that the first speed is very critical for the damage accumulation.

From the engineering side, this will require further investigation, at is seems that the number of cycles might be overestimated.

3.7.2 COMPARISON OF SAFETY VALUES

The safety values for pitting are comparable between ISO and AGMA. When the duty cycle is applied, the safety values are little higher than for the one-bin nominal load (Figure 20, left). Comparing the safety factors for root bending shows that the results for AGMA are clearly lower than for ISO (Figure 20, right). The safety results of the AGMA standard are more conservative, which may lead to over-dimensioned gears.

4 CONCLUSION

The design process of a bevel gear set is challenging. The rating of bevel gears is available through standards or through the LTCA method. Both methods are suitable for a certain step in the design phase. The most efficient process is achieved only when combining both the rating methods from standards together with LTCA.

The ISO standard allows the engineer to adjust some parameters to achieve good comparability with the LTCA. The AGMA standard also has some (but fewer) adaption possibilities. One major difference between the AGMA standard and the ISO standard is the much lower root bending stress, which is recommended to be adjusted to match the LTCA and ISO results.

The development of the contact pattern is a major step in the design and optimization phase of the bevel gearset. Therefore, the misalignment values from the transmission simulation are required. The contact pattern development has to satisfy both the unloaded and the loaded contact. From the contact pattern development and the tool design in GEMS, the effective face width and the blade tip radius have to be taken over into the rating standard calculation, to increase the accuracy of the rating standard calculation.

The rating calculation allows a quick rating of macro geometry variants, including duty cycles. The safety numbers show quite comparable results for the pitting calculation. For the root stress, the AGMA seems to be rather conservative for bevel gears.

Overall, the combination of the drivetrain design software and LTCA using the FE approach allows a complementary calculation for the bevel gear rating. This establishes a complete and very reliable process for the development of bevel and hypoid gearsets.

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